Performance of Low-Feedback-Rate, Gradient-Based OFDMA Subcarrier Allocation with Partial Channel Information

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Abstract—We consider a downlink OFDMA subcarrier allocation problem where the allocation algorithms are gradient-based but only CSI for some subset of subcarriers from each user is available at the transmitter. We propose an iterative version of gradient-based OFDMA subcarrier allocation schedulers which performs better than the standard non-iterative gradient-based schedulers. In addition, we propose an adaptive, weighted M-Best feedback mechanism (called wM-Best) suitable for gradient-based schedulers than the standard M-Best feedback. Simulation results show improvement of system performance metrics, in terms of average throughput and queue length (or equivalently average delay), of the iterative scheduler and also the wM-Best feedback.

I. INTRODUCTION

In OFDMA downlink systems, it is well known that if the base station knows the channel gain or fading of every subcarrier for every user, the subcarriers can be allocated adaptively to different users in order to exploit multi-user diversity and hence improving the system performance. Having perfect channel knowledge (or channel state information (CSI)) at the transmitter, adaptive subchannel schemes achieve very good performance [1].

However, in practical systems, a perfect channel knowledge is hardly available at the transmitter due to the large amount of feedback overheads. For example, for a single-antenna system with $K$ users and $N$ subcarriers, there are $KN$ real-valued CSI feedback values that need to be reported every scheduling period. The overhead will be further multiplied in multi-antenna (MIMO) OFDMA system. A number of feedback reduction schemes have been proposed. For example, the feedback CSI may be quantized [2], multiple subcarriers may be grouped into a subchannel and only one bit of CSI is reported for every subcarrier in the subchannel [3], and each user only reports its $M$ best subcarriers (called M-Best feedback) [4].

A important observation to reduce the feedback overhead is to opportunistically feedback only from the users that are most likely to be allocated the resources [5]. In this paper, we introduce a partial CSI feedback scheme called Weighted M-Best (wM-Best), which is a modified version of the M-Best feedback, where each user feed backs the CSI of some number of its best subcarriers. The number of CSI feedbacks a user reports depend on the CSI weighted by the queue length (queue state information, QSI) of the user at the base station. These weights are also used by the scheduler to allocate the OFDM subcarriers to users.

In this paper, we consider gradient-based subcarrier schedulers which assign subcarriers or equivalently select the transmission rate of each user that maximizes the projection onto the gradient of the system’s total utility [6]–[8]. The gradient-based schedulers cover a general range of algorithms ranging from proportionally-fair (PF) algorithms to throughput-optimal algorithms such as Max-Weight [9]. In addition to a comparison of existing gradient-based schedulers, we also propose a new gradient-based subcarrier allocation algorithm which is an iterative version of the existing algorithms. Shown by simulation, our proposed algorithm performs better than the existing (non-iterative) gradient-based schedulers since the utility weights are not updated as subcarriers are being assigned. The iterative concept of gradient-based algorithms is taken from the iterative algorithms proposed in [10]–[12]. Our proposed algorithm is adapted from the iterative subcarrier allocation algorithm proposed in [11] but we include consideration of utility weights in the allocation. The algorithm in [11] can be considered as two-dimensional allocation scheme, where the algorithm finds the highest channel-gain both in row and column. However, the algorithm does not include the queue information in the allocation decisions.

In summary, the contribution of this paper is as following:

- Propose an adaptive, weighted M-Best feedback (called wM-Best) scheme which works well with gradient-based subcarrier allocation algorithms.
- Propose an iterative gradient-based scheduler which allocates subcarriers iteratively based on CSI and QSI while updates the queue-length weights in the subcarrier allocation algorithm.
- Show by simulation the improved performance of the proposed wM-Best feedback scheme and the iterative subcarrier allocation algorithm with the reference case of perfect CSI knowledge and the standard M-Best feedback scheme and the non-iterative gradient-based schedulers.

The rest of this paper is organized as follow. The problem formulation is described in Section II. Detailed descriptions
of the various feedback types and gradient-based resource allocation algorithms are also presented in this section. Section III shows the results of our performance evaluation. Finally, Section IV concludes the paper.

II. PROBLEM FORMULATION

Consider a downlink single-hop OFDMA system composed of a base station (BS) and $K$ users. There are $N$ OFDM subcarriers. Time is divided into TDM timeslots. The users are homogeneous, i.e. they see statistically symmetric arrival and channel processes and have the same priority. Packets of fixed size arrive stochastically for each user. To accommodate the randomness in the arrival streams, there are $K$ queues at the base station, one for each user, to buffer the data. Each user has an infinite buffer to store the data packets that cannot be immediately transmitted. At the beginning of each timeslot, the assignment of subcarriers to users is made by a centralized resource manager at the base station. The resource manager has perfect knowledge of the queue backlogs but a partial knowledge of the channel states. We do not allow sharing of any subcarriers. The assignment is announced immediately to all users via a separate control channel. In this paper, we assume that the transmit power of the base station is allocated evenly across all OFDM subcarriers, i.e., we assume no water-filling-type power loading. As a result of equal power loading, random fading channel conditions can be mapped into a matrix of rates. As in [13], this OFDMA subcarrier allocation problem with stochastic arrivals and channels can be modeled as a multi-queue multi-server scheduling problem as shown in Figure 1 where the time-varying rate matrix is the service capacity matrix of the $K$ servers.

We use the following notation: at the beginning of timeslot $t$, the queue length or backlog (in packets) and the running-average throughput up to timeslot $t$ of user $i$ are denoted by $Q_i(t)$ and $W_i(t)$, respectively. During timeslot $t$, the number of packets arrival to queue $i$ is $a_i(t)$ and the number of packets that could be served by subcarrier $j$ from user $i$ is $r_{ij}(t)$. The assignment variable $s_{ij}(t) = 1$ means subcarrier $j$ is assigned to user $i$ and $s_{ij}(t) = 0$ otherwise. Note that $\sum_{j=1}^{K} s_{ij} \leq 1$ for all $j = 1, \ldots, N$ since sharing of subcarriers is not allowed.

With some abuse of notation, we define the total feasible rate (or number of served packets) of user $i$ during timeslot $t$ as $r_i(t)$, where

$$r_i(t) := \sum_{j=1}^{K} s_{ij}(t)r_{ij}(t).$$

(1)

The dynamics of the queue length for queue $i$ are given as

$$Q_i(t+1) = [Q_i(t) - r_i(t)]^+ + a_i(t),$$

(2)

where $[x]^+ = \max \{0, x\}$ for any real $x$. In addition, the dynamics of the running-average throughput of user $i$ are given as

$$W_i(t+1) = \theta W_i(t) + (1 - \theta) \min (Q_i(t), r_i(t)),$$

(3)

where the constant $\theta \in [0,1]$ specifies how much the running-average depends on the current throughput.

In this paper, we consider an i.i.d. traffic arrival model across users and timeslots. Specifically, the number of fixed-size packets arriving to user $i$ during timeslot $t$, $a_i(t)$, is modeled as an i.i.d. Poisson random variable with mean $\lambda$ packets/timeslot.

A. Channel Model

To determine the rate matrix $[r_{ij}]$, we assume a block-fading channel where for each user the gains of the OFDM subcarriers are independent over timeslots but, within the same timeslot, the gains may be correlated. Specifically, we assume an $L$-tap delay line channel model [14]. For each user, the channel consists of $L = [BW/\Delta f_c]$ independent resolvable paths where $BW$ and $\Delta f_c$ are the total bandwidth and the coherence bandwidth of the system (assume the same for all users for simplicity), respectively, and each path has equal power. The channel impulse response between the base station and user $i$ is $h_i(t) = \sum_{l=0}^{L-1} h_{il}\delta(t - l/BW)$ where $[h_{il}]$ are modeled as independent identically distributed (i.i.d) circularly symmetric complex Gaussian random variables with distribution $CN(0,1/L)$. For each user $i$, the $N$-dimensional subcarrier gains $[H_{ij}(t)]$ are calculated from the $N$-point FFT of the channel impulse response $h_i(t)$.

We assume that the base station has a maximum transmit power of $P$ and divides this power equally among all $N$ subcarriers. With perfect channel estimation at the receiver, the feasible number of packets that could be served from queue $i$ by subcarrier $j$ during timeslot $t$ is related to the signal-to-noise ratio and given as

$$r_{ij}(t) = \frac{BW}{N} \log_2 \left( 1 + 0.56 \frac{P}{N} |H_{ij}(t)|^2 \right),$$

(4)

where $\gamma$ is the ratio of the number of OFDM symbols/timeslot over the packet size (in bits) and we assume the noise power is normalized to be one for simplicity and the constant coefficient 0.56 accounts for the SNR gap according to practical modulation and coding limitations [7].
B. Gradient-Based Subcarrier Allocation Scheduling Algorithms

The subcarrier allocation algorithms are based on the gradient-based scheduling frameworks, which allocate channels that maximize the projection onto the gradient of the system’s total utility. The gradient-based schedulers are selected since they include scheduling algorithms ranging from proportionally fairness, Max-SNR, to Max-Weight algorithms.

Specifically, the system’s total utility function is given as $U(W(t)) := \sum_{i=1}^{K} U_i(W_i(t))$, where $U_i(W_i(t))$ is an increasing concave utility function of user $i$’s running-average throughput $W_i(t)$ up to time $t$. An example of utility functions is

$$U_i(W_i(t)) = \begin{cases} \frac{\alpha}{\alpha} (W_i(t))^{\alpha}, & \alpha \leq 1, \alpha \neq 0, \\ c_i \log(W_i(t)), & \alpha = 0, \end{cases}$$

where $\alpha \leq 1$ is the fairness parameter and $c_i$ is the QoS weight [7]. For this utility function, the gradient-based scheduler select the rate vector which is the solution to

$$\max_{[r_1(t),...,r_K(t)] \in R_i} \sum_{i=1}^{K} c_i(W_i(t))^{\alpha-1}r_i(t),$$

where $R_i$ is the feasible rate region given the reported channel-gains at timeslot $t$. Note that setting $\alpha = 1$ results in a scheduling rule that maximizes the total throughput during each timeslot (i.e., the Max-SNR rule), while setting $\alpha = 0$ results in the proportionally fair rule. Other intermediate values of $\alpha \in (0, 1)$ provides a trade off between total throughput and fairness. As stated in [7], the gradient-based algorithms can be generalized to the problem of

$$\max_{[r_1(t),...,r_K(t)] \in R_i} \sum_{i=1}^{K} \mu_i(t)r_i(t),$$

where $\mu_i(t)$ is the time-varying weight assigned to user $i$ at time $t$. This weight can be the gradient of the running-average throughput as given above or can be a function of the queue length and/or delay such as in the Max-Weight algorithms [9].

Due to the multi-server (or multi-subcarrier) nature of the OFDMA subcarrier allocation problems, the rate $r_i(t)$ of user $i$ during timeslot $t$, as given in (1), is a composition of the rates from all its assigned servers. Furthermore, within the same timeslot, the weight of user $i$ which depends on its queue length changes as the scheduler assigns it a server. As will be shown by simulation, ignoring this fact in the gradient-based schedulers results in too many servers assigned to the users with long queues at the beginning of the timeslot. This results in smaller total throughput and hence less multi-user diversity gain.

Hence, in our study, we are interested in evaluating the performance of the following two versions of gradient-based schedulers: non-iterative and iterative and with different values of the fairness parameter $\alpha$ and partial CSI feedback schemes which will be described later.

1) Non-Iterative Gradient-Based Schedulers: These schedulers are the gradient-based schedulers that solve (7). In this paper, we divide the algorithms into two types depending on the value of $\alpha$ and the definition of the weight $\mu_i(t)$. Specifically, we let the weight $\mu_i(t)$ of user $i$ in (7) be

$$\mu_i(t) = \begin{cases} (W_i(t))^{\alpha-1}, & \alpha \in [0, 1], \\ (Q_i(t))^{\alpha-1}, & \alpha > 1, \end{cases}$$

where we assume $c_i = 1$ for simplicity and $W_i(t)$ and $Q_i(t)$ are the running-average throughput and the queue length of user $i$, respectively, at the beginning of the timeslot $t$. These two values are not updated as subcarriers are being assigned to users. Note that by changing the dependency of the weights on either running-averages or queue lengths, we have gradient-based algorithms (when $\alpha \in [0, 1]$) described in [7] and queue-based max-weight algorithms (when $\alpha > 1$), respectively.

Since the weights $\mu_i(t)$ for all $i$ stay constant during allocation of all subcarriers, the subcarrier allocation can be decomposed into a sequential allocation where the scheduler allocates each subcarrier one by one in any ordering and subcarrier $j$ is allocated to the user with the largest value of $\mu_i(t)r_{ij}(t)$, where ties are broken randomly.

2) Iterative Gradient-Based Scheduler: As observed earlier, for the max-weight algorithms which use queue lengths as weights, not adjusting the queue lengths as users are assigned subcarriers should result in over assignment to long queues and a degradation in the system performance. Hence, we propose a heuristic but iterative version of the gradient-based schedulers where subcarriers are allocated one by one and the queue lengths are updated as subcarriers are assigned. Specifically, consider an example where the scheduler has just assigned subcarrier $j$ to user $i^*$ who has the largest value of $Q_i(t)r_{ij}(t)$. Then the scheduler updates the queue length of user $i^*$ to $[Q_{i^*}(t) - r_{i^*j}(t)]^+$ and then continues allocating another unassigned subcarrier with this new value of $Q_{i^*}(t)$.

However, the performance of this heuristic algorithm depends on the order of which subcarriers are allocated first.

In this paper we propose the following heuristic algorithm which is adapted from the two-dimensional subcarrier allocation proposed [11] which considers only CSI. Our algorithm takes into account both the queue lengths (QSI) and channel (CSI) knowledge. Note that other iterative OFDMA subcarrier allocations based on CSI and QSI have also been proposed [10], [12].

Iterative Gradient-Based Scheduler:

Initialize: $s_{ik} = 0$ for all $i$ and $k$, $V = \{1, \ldots, N\}$, $G = 0$, and $j = 1$.

1) Given the weighted-rate matrix $[Q_{r_{ij}}]$ and subcarrier $j$, select user $i^*$ that has the largest (non-zero) value of $Q_{i^*r_{ij}}$, where ties are broken randomly.

2) Check whether $Q_{i^*r_{i^*j}}$ is the largest among all $Q_{i^*r_{i^*k}}$ for $k \in V$.

   a) If yes, assign subcarrier $j$ to user $i^*$ (i.e., set $s_{i^*j} = 1$). Update $V \leftarrow V \setminus \{j\} \cup G$, $G \leftarrow \emptyset$, and $Q_{i^*} \leftarrow$
The updated queue lengths and the queue length of the user is updated to be
\[ Q_{i}^{*} - r_{i,j} \]. Update the \([Q_{i}r_{ij}]\) matrix with the
new value of \(Q_{i}^{*}\).

b) If no, update \(V \leftarrow V - \{j\}\) and \(G \leftarrow G \cup \{j\}\).

3) Pick the first subcarrier \(j\) in \(V\) and repeat steps 1) and 2) until either \(V = \emptyset\) or \(Q_{i}r_{ik} = 0\) for all \(i = 1, \ldots, K\) and for all \(k \in V\).

In the above algorithm, the set \(V\) keeps track of unassigned subcarriers that have not been attempted at allocation while the set \(G\) keeps track of the unassigned subcarriers which have been attempted at allocation. \(G\) is re-initialized to an empty set every time an unassigned subcarrier is successfully assigned. The concept of the above heuristic algorithm is that it attempts to find the subcarrier \(j^{*}\) and user \(i^{*}\) which has the largest value of \(Q_{i}r_{ij}\) among all unassigned subcarriers or while the queues are not all zero, where \(Q_{i}\) is updated each time a subcarrier is assigned.

An example of the iterative gradient-based scheduler with 3 users and 3 subcarriers can be illustrated as follows:

Initial Parameters:
\[
\begin{bmatrix}
Q_{1} = [5.2 \ 4.4 \ 4.5] , \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
r_{ij} = \begin{bmatrix}
1.0 & 0.9 & 1.3 \\
1.7 & 0.4 & 0.6 \\
1.8 & 1.1 & 0.7 \\
\end{bmatrix} \end{bmatrix} \quad \text{and} \quad [Q_{i}r_{ij}] = \begin{bmatrix}
5.2 & 4.7 & 6.8 \\
7.5 & 1.8 & 2.6 \\
8.1 & 5.0 & 3.2 \\
\end{bmatrix}
\]

Iteration 1: Start with subcarrier 1. It is assigned to user 3 and the queue length of the user is updated to be \(4.5 - 1.8 = 2.7\). The updated queue lengths and \([Q_{i}r_{ij}]\) matrix (with the first column removed since subcarrier 1 has been assigned) are

\[
\begin{bmatrix}
Q_{i} = [5.2 \ 4.4 \ 2.7] \quad \text{and} \quad [Q_{i}r_{ij}] = \begin{bmatrix}
-4.7 & 6.8 \\
-1.8 & 2.6 \\
-3.0 & 1.9 \\
\end{bmatrix}
\end{bmatrix}
\]

Iteration 2: Start with subcarrier 2. User 1 has the highest \(Q_{1}r_{12}\) at 4.7 which is less than \(Q_{1}r_{13} = 6.8\). Hence, the scheduler moves to consider subcarrier 3 which is the only subcarrier in \(V\). Subcarrier 3 is then assigned to user 1. The updated queue lengths and \([Q_{i}r_{ij}]\) matrix (with the first and third columns removed since subcarrier 3 has just been assigned) are

\[
\begin{bmatrix}
Q_{i} = [3.9 \ 4.4 \ 2.7] \quad \text{and} \quad [Q_{i}r_{ij}] = \begin{bmatrix}
-3.5 & - \\
-1.8 & - \\
-3.0 & - \\
\end{bmatrix}
\end{bmatrix}
\]

Iteration 3: Start with subcarrier 2, which is the only unassigned subcarrier. Assign it to user 1.

Hence, for the iterative scheduler the subcarrier allocation matrix and the queue lengths at the end of the current timeslot are

\[
\begin{bmatrix}
s_{ij} = \begin{bmatrix}
0 & 1 & 1 \\
0 & 0 & 0 \\
1 & 0 & 0 \\
\end{bmatrix} \quad \text{and} \quad Q_{i} = [3.0 \ 4.4 \ 2.7].
\end{bmatrix}
\]

Comparing these results with those under non-iterative scheduler, we see that the allocation matrix and the queue lengths at the end of the current timeslot are

\[
\begin{bmatrix}
s_{ij} = \begin{bmatrix}
0 & 0 & 1 \\
0 & 0 & 0 \\
1 & 1 & 0 \\
\end{bmatrix} \quad \text{and} \quad Q_{i} = [3.9 \ 4.4 \ 1.6].
\end{bmatrix}
\]

Note that it is demonstrated via this example that the algorithm does terminate and reach an allocation.

C. CSI Feedback

In this paper we study the performance of several subcarrier allocation algorithms with a partial CSI knowledge. Specifically, we consider the situation where the channel gains for all subcarriers and users are estimated perfectly but only CSI of some selected subcarriers from each user are fed back to the BS. We consider three types of feedback mechanisms: full CSI feedback, M-Best feedback, and weighted M-Best feedback.

1) Full CSI feedback: This provides the best performance compared with the others and hence provides a benchmark for comparison. However, the feedback overhead is relatively high since we require \(NKB_{real}\) bits per timeslot where \(B_{real}\) is the number of bits required to quantize a real number with negligible quantization error.

2) M-Best Feedback (M-Best): Each user feeds back via an uplink feedback timeslot to the BS only its \(M\) highest channel-gains and their corresponding subcarrier numbers, where \(M\) is constant for all users and timeslots \([4]\). We assume that any subcarrier \(j\) of user \(i\) whose gain is not reported has rate \(r_{ij} = 0\), while the reported subcarrier has the rate given in (4).

3) Weighted M-Best Feedback (wM-Best): Since the gradient-based schedulers allocate subcarriers based on not only the channel-gains but the running-average throughputs or queue lengths as well, this means users with large queue lengths or running-average throughputs are likely to be allocated more subcarriers and hence they should report more number of their best subcarriers. This observation motivates an adaptive version of the M-Best feedback mechanism, called wM-Best, where we should adapt the value of \(M\) depending on the current weights specified by the scheduler. That is, the user \(i\) at the beginning of timeslot \(t\) reports its \(M_{i}(t)\) best subcarriers where

\[
M_{i}(t) = \left\lfloor \frac{\mu_{i}(t)}{\sum_{k=1}^{K} \mu_{k}(t)} \cdot KM \right\rfloor.
\]
Figure 2. Comparison of the average throughput and average queue length for different schedulers with full CSI feedback.

full and partial CSI feedback mechanisms. For the non-iterative schedulers, we consider the schedulers with $\alpha = 0, 0.5, 1, 2, \text{ and } 4$. Recall that the scheduler is proportionally fair when $\alpha = 0$, Max-SNR when $\alpha = 1$, and MaxWeight when $\alpha > 1$. The case of $\alpha = 0.5$ provides an intermediate scheduler that balances fairness and throughput. To compare the performance of the iterative scheduler over non-iterative scheduler, we consider only the MaxWeight scheduler when $\alpha = 2$.

The simulation is performed with $N = 64$ subcarriers and $K = 64$ users under 200 timeslots. We use the $L$-tap delay line channel with $L = 10$. The average received SNR per subcarrier is 10 dB. Packet size and OFDM parameters are selected such that the system capacity is about 9 packets/user/timeslot. The first simulation result compares the average throughput and average queue length for different non-iterative and iterative schedulers to show the improved performance of the iterative scheduler with full CSI. The second simulation result shows the dependency of the performance on the value of $M$ in the M-Best and wM-Best feedbacks.

Figure 2 also shows that, among the non-iterative running-average-throughput-based schedulers (i.e., when $\alpha = 0, 0.5, 1$), the scheduler with $\alpha = 0.5$ surprisingly shows a better performance than the PF or Max-SNR schedulers than all non-iterative schedulers, especially when the arrival rate is high. The improved performance can be traced back to the fact that the iterative scheduler allocates subcarriers to many more number of users, as shown in Figure 3 which shows the average and standard deviation of the number of users who are assigned some subcarriers. At low packet arrival rates, the iterative scheduler serves almost every user while the non-iterative schedulers serve less than 20 users. Hence, the non-iterative schedulers over assign subcarriers to some users who may not have enough data in the queues to send over those assigned subcarriers. This results in a waste of subcarrier capacity and hence reduced system performance. This confirms the intuition provided in [12] that, to have small queues and hence small delays, the scheduler should serve as many users as possible during each timeslot.

Figure 3. The average and standard deviation of the number of assigned users in each time slot.
(\(\alpha = 0\) and 1, respectively) at the arrival rates less than the system capacity (i.e., for \(\lambda \leq 9\)). This shows that, with the non-iterative weights, concerning too much on fairness like in PF scheduler (i.e., making sure that every user has similar running-average throughput) or too much on the sum throughput degrades the system performance compared to the scheduler that balances fairness and throughput.

Next, we show the improved performance of wM-Best feedback over M-Best feedback, under iterative and non-iterative MaxWeight schedulers (i.e., \(\alpha = 2\)). Figure 4 shows the average throughputs against different values of \(M\) where \(\lambda = 7.5\). The figures show that the performance with the wM-Best feedback is better than that with M-Best feedback. Note that given a feedback scheme there is very little performance difference between non-iterative and iterative schedulers. The reason could be seen from Figure 5 which shows the average number of served users for all schedulers with wM-Best feedback. Let us consider only the \(\alpha = 2\) schedulers. At low values of \(M\) the iterative and non-iterative schedulers serve about the same number of users on average. Although the non-iterative scheduler serves less users as \(M\) gets larger, there is enough number of CSI reports (i.e., \(KM\) CSI values) so that the system performance is not affected by the non-adaptiveness of the weights \([Q_i(t)]\).

IV. CONCLUSIONS

In this paper, we propose an iterative gradient-based OFDMA subcarrier allocation scheduler which, as shown by simulation, performs better than the standard non-iterative gradient-based scheduler. In addition, we propose the wM-Best feedback mechanism suitable for gradient-based schedulers than the standard M-Best feedback. Simulation results show improvement of system performance metrics, in term of average throughput and queue length (or equivalently average delay), of the iterative scheduler and the wM-Best feedback.

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