# Sirindhorn International Institute of Technology

## Thammasat University at Rangsit

School of Information, Computer and Communication Technology

# TCS 455: Problem Set 5

## Due date: Feb 5, 2010 (Friday)

## Instructions

1. ONE part of a question will be graded. Of course, you do not know which problem will be selected; so you should work on all of them.
2. Late submission will not be accepted.
3. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Please submit your solutions for all of the following questions.

1. Fourier transform review

    a. Give a **simplified** expression for the Fourier transform $C(f)$ of a waveform $c(t)$ when

    $$c(t) = \begin{cases} A, & 0 \le t < T \\ 0, & \text{otherwise} \end{cases}$$

    b. A message $m = (m_0, m_1, m_2, m_3) = (1, -1, 1, 1)$ is sent via

    $$s(t) = \sum_{k=0}^{\ell-1} m_k c(t - kT) \text{ where } \ell \text{ is the length of } m.$$

    Find a **simplified** expression for the Fourier transform $S(f)$ of the waveform $s(t)$.

c. Assume T = 2 [ms] and A = 1 [mV]. Find $S(0)$ and **plot** $|S(f)|$ where $f \in [-2, 2]$ kHz for the following $m$.

    i.   $m = (1)$

    ii.   $m = (1,1)$

    iii.   $m = (1,1,0,0)$

    iv.   $m = (1,1,-1)$

    v.   $m = (1,1,-1,1)$

    vi.   $m = (1,1,-1,-1)$

    vii.   $m = (1,1,-1,1,-1,-1,1,1,1,-1,1,1)$

d. What did you learn from the plots in part (c)?

2. Consider Global System for Mobile (GSM), which is a TDMA/FDD system that uses 25 MHz for the forward link, which is broken into radio channels of 200 kHz. If 8 speech channels are supported on a single radio channel, and if no guard band is assumed, find the number of simultaneous users that can be accommodated in GSM.

3. Draw the complete state diagrams for linear feedback shift registers (LFSRs) using the following polynomials. Does either LFSR generate an m-sequence?

    a.   $1 + x^2 + x^5$

    b.   $1 + x + x^2 + x^5$

    c.   $1 + x + x^2 + x^4 + x^5$

4. Use any resource, find **all** primitive polynomials of degree 6 over GF(2). Indicate your reference.

5. We have seen in class that the following waveforms are orthogonal.
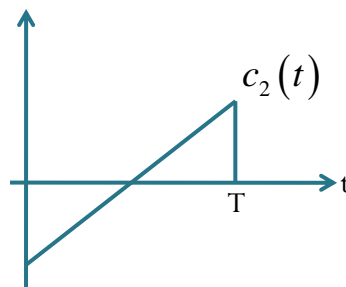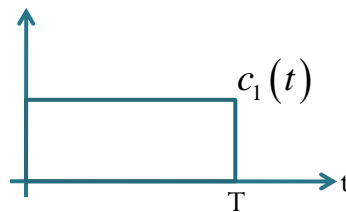
Suppose we want to have four orthogonal waveforms on [0,T]. Find two more waveforms, $c_3(t)$ and $c_4(t)$, that are time-limited to [0,T]. Make sure that all four waveforms are mutually orthogonal.

6. In CDMA, each bit time is subdivided into $m$ short intervals called **chips**. We will use 8 chips/bit for simplicity. Each station is assigned a unique 8-bit code called a **chip-sequence**. To transmit a 1 bit, a station sends its chip sequence. To transmit a 0 bit, it sends the one's complement[1] of its chip sequence.

Here are the binary chip sequences for four stations:

   A: 0 0 0 1 1 0 1 1
   B: 0 0 1 0 1 1 1 0
   C: 0 1 0 1 1 1 0 0
   D: 0 1 0 0 0 0 1 0

For pedagogical purposes, we will use a bipolar notation **with binary 0 being -1 and binary 1 being +1**. In which case, during each bit time, a station can transmit a 1 by sending its chip sequence, it can transmit a 0 by sending the negative of its chip sequence, or it can be silent and transmit nothing. We assume that all stations are synchronized in time, so all chip sequences begin at the same instant.

When two or more stations transmit simultaneously, their bipolar signals add linearly.

   a. Suppose that A, B, and C are simultaneously transmitting 0 bits. What is the resulting (combined) bipolar chip sequence?

   b. Suppose the receiver gets the following chips: (-1 +1 -3 +1 -1 -3 +1 +1).
      Which stations transmitted, and which bits did each one send?

   c. One of your friends wants to work on part (a) and (b) using MATLAB. Here is his code with two incomplete lines.

```
%Chip sequences
C = [0 0 0 1 1 0 1 1; 0 0 1 0 1 1 1 0; 0 1 0 1 1 1 0 0; 0 1 0 0 0 0 1 0];
C = 2*C-1; %Change to bipolar form

% Part a
m = [-1 -1 -1 0] %message to transmit
x = %%%%%%%%%HELP ME%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Part b
r = [-1 1 -3 1 -1 -3 1 1] ;
m_decoded = 1/8* %%%%%%%%%HELP ME%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%This gives [mA mB mC mD]' in bipolar form;
%The value is 1 if 1 was transmitted. The value is 0 if nothing was
%transmitted. The value is -1 if 0 was transmitted.
```

   Help him find the expression for "x" and "m_decoded" in the code above. Note that the expression for x should be in terms of C and m. The expression for m_decoded should be in terms of C and r.

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[1] You should have seen the "one's complement" operation in your "digital circuits" class.