Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

TCS 455: Problem Set 3

Semester/Year: **2/2009**
Course Title: Mobile Communications
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Course Web Site: http://www.siit.tu.ac.th/prapun/ecs455/

Due date: Dec 18, 2009 (Friday)

Instructions

1. ONE part of a question will be graded. Of course, you do not know which problem will be selected; so you should work on all of them.
2. Late submission will not be accepted.
3. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Please submit your solutions **along with your MATLAB codes** for all of the following questions. You may use the MATLAB code from

http://infohost.nmt.edu/~borchers/erlang.html

to evaluate the Erlang B formula.

1. In this question, we will explore the Poisson process using discrete time approximation as discussed in class. We consider a Poisson process from time 0 to time T = 1000. The time unit is irrelevant in this simulation. However, to make the question explicit, we will take the time unit to be in hour. The arrival rate (or the call request rate) is $\lambda = 30$ arrivals per hour. By using the discrete time approximation, we will divide the time interval into n = 1,000,000 slots.

a. As shown in class, the number of arrivals in the slots can be approximated by i.i.d. Bernoulli random variables with probability p_1 of having exactly one arrival. Find this p_1.

b. Generate all *n* Bernoulli random variables simultaneously using the command:

```
pp = binornd(1,p1,1,n)
```

c. Let N_k be the total number of arrivals during the interval $\left[\frac{k-1}{m}T, \frac{k}{m}T\right)$ where *k* = 1, 2, …, *m*. In this case, let *m* = 1,000. As shown in class, the random variables $N_1, N_2, \ldots, N_m$ are i.i.d. Poisson random variables.

   i. What is the mean (expected value) of these Poisson random variables?

   ii. Use MATLAB's `poisspdf` function to calculate the probability that $N_1 = 30$.

   iii. Use MATLAB's `poisspdf` function to calculate the probability that $N_{30} = 30$.

   iv. What is the probability that $N_{50} = 10.5$?

d. We can approximate $P[N_1 = 30]$ by calculating the frequency of occurrence from the i.i.d. $N_1, N_2, \ldots, N_m$.

   i. Explain how

   ```
   N = sum(reshape(pp,n/m,m))
   ```

   gives $N_1, N_2, \ldots, N_m$.

   ii. Let A be the number of $N_1, N_2, \ldots, N_m$ that take the value 30. Find *A*.

   iii. The frequency of occurrence is $\frac{A}{m}$. Compare your $\frac{A}{m}$ with the answer from part c.ii and c.iii.

e. The numbers of slots between the adjacent arrivals (1's) in the discrete-time approximation model are given by

   `diff(find(pp==1))` [slots].

   The corresponding continuous time durations between adjacent arrivals are given by

   `W = diff(find(pp==1))*T/n` [hrs].

   These time durations are called inter-arrival times. *Use frequency of occurrence* to approximate the probability that the inter-arrival time will be greater than 2 minutes.

f. As shown in class, the inter-arrival times $W_1, W_2, W_3, \ldots$ are i.i.d. exponential random variables. Use MATLAB's `expcdf` function to calculate $P[W_{10} > 2 \text{ min}]$. Compare your answer with the answer in the previous part.

2. In this question, we will explore the relationship between exponential random variable and geometric random variable.
   a. Start with an exponential random variable X whose mean is $\frac{1}{\mu}$. What is its pdf?
   b. What is the probability that X is in the interval $[a,b)$?
   c. What is the probability that X is in the interval $[(k-1)T,kT)$? Assume T is a positive real number and k is a positive integer. We will denote this probability by $P\left[X \in [(k-1)T,kT)\right]$.
   d. Consider the sequence of number $p_1, p_2, p_3, \ldots$ where $p_k = P\left[X \in [(k-1)T,kT)\right]$. Are these p_k's agrees with a pmf of a geometric random variable? Note that the pmf of a geometric random variable can be expressed as $(1-r)r^{k-1}$. Can you find r such that the $p_1, p_2, p_3, \ldots$ above satisfies the formula $(1-r)r^{k-1}$?
3. In this question, we will explore the Erlang B formula using the discrete time approximation. We will use the same Poisson process from the first question (Q1).
   a. Find the total number of arrivals during the time 0 to time T = 1000. Denote this number by V. What is the your value of V?
      This means there will be V call requests during our time interval of interest.
   b. For each call request in the previous part, we want to find the call durations. These durations are assumed to be i.i.d. exponential random variables with mean $\frac{1}{\mu}$ = 2 min. So, we can generate all of the call durations by

      ```
      exprnd(2/60,1,V)
      ```

      We want to convert these numbers into numbers of slots. So, we find

      ```
      D = ceil(exprnd(2/60,1,V)/(T/n));
      ```

      i. Express the pmf of D. Approximate the parameter of this pmf.
         Hint: Use question 2 and the fact that $e^{-x} \approx 1-x$ when x is small.
      ii. What is the expected value of D?
   c. (Difficult) Suppose there are a total of m = 2 channels for this system.
      i. Out of the V call requests, count the number B of blocked calls.
      ii. Compare the number $\frac{B}{V}$ with the blocking probability from the Erlang B formula.
4. What did you learn from this assignment?