

Practice Questions for “Induction and Inductance”

- [HRW, 9E, P30.40] The inductance of a closely packed coil of 400 turns is 8.0 mH. Calculate the magnetic flux through the coil when the current is 5.0 mA.
Ans: 10^{-7} Wb
- [HRW, 9E, P30.44] A 12 H inductor carries a current of 2.0 A. At what rate must the current be changed to produce a 60 V emf in the inductor?
Ans: -5 A/s
- [HRW, 9E, P30.45] At a given instant the current and self-induced emf in an inductor are directed as indicated in Figure 1. (a) Is the current increasing or decreasing? (b) The induced emf is 17 V, and the rate of change of the current is 25 kA/s; find the inductance.

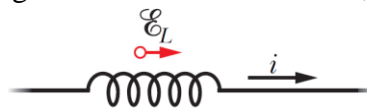


Figure 1: [HRW, 9E, Fig. 30-57]

- Ans: (a) decreasing (b) 6.8×10^{-4} H.
- A conducting loop of area 240 cm^2 and resistance 12Ω lies at right angles to a spatially uniform magnetic field. The loop carries an induced current of 320 mA. At what rate is the magnetic field changing?
Ans: 160 T/s
 - A 60 mA current is flowing in a 100 mH inductor. Over a period of 1.0 ms the current is reversed, going steadily to 60 mA in the opposite direction. What is the inductor emf during this time?
Ans: 12V
 - The current in a 2.0 H inductor is given by $i(t) = 3t^2 + 15t + 8$ where t is in seconds and $i(t)$ is in amperes. Find an expression for the magnitude of the inductor emf.
Ans: $12t + 30$ V
 - [Modified from HRW, 9E, P30.53] A solenoid having an inductance of $6.30 \mu\text{H}$ is connected in series with a $1.20 \text{ k}\Omega$ resistor. A 14.0 V battery is connected across the pair as shown in Figure 2. Suppose, at time $t_0 = 0$ s, the current i through the resistor is 0 A.

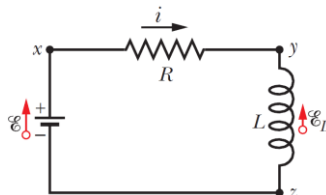


Figure 2: [HRW, 9E, Fig. 30-16]

- A long time later ($t \rightarrow \infty$), what is the value of i . $i_\infty = \frac{\mathcal{E}}{R} \approx 11.7 \text{ mA}$
- Find the inductive time constant τ_L .

$$\tau_L = \frac{L}{R} \approx 5.25 \text{ ns}$$

Additional Practice Questions for “Induction and Inductance”

8. [Modified from HRW, 9E, P30.53] (Continued from Q7) A solenoid having an inductance of $6.30 \mu\text{H}$ is connected in series with a $1.20 \text{ k}\Omega$ resistor. A 14.0 V battery is connected across the pair as shown in Figure 2. Suppose, at time $t_0 = 0 \text{ s}$, the current i through the resistor is 0 A .
- Find the current i at an arbitrary time $t > 0 \text{ s}$. (This will be a function of t .)
 - Find the current i at time $t = 0.000000001 \text{ s}$.
 - Find the time at which the current through the resistor reaches 80.0% of its final value.
 - What is the current through the resistor at time $t = t_0 + \tau_L$?

Ans: ~~(1) 11.7 mA, (b) 5.25 ns~~, (a) $i(t) = 11.7(1 - e^{-1.9 \times 10^8 t}) \text{ mA}$; ~~(b) 2.0 mA; (c) 8.45 ns;~~ (d) 7.38 mA .

9. [HRW, 9E, P30.91] In the circuit of Figure 3, $R_1 = 20 \text{ k}\Omega$, $R_2 = 20 \text{ k}\Omega$, $L = 50 \text{ mH}$, and the ideal battery has $\mathcal{E} = 40 \text{ V}$. Switch S has been open for a long time when it is closed at time $t = 0$. Just after the switch is closed, what are (a) the current i_{bat} through the battery and (b) the rate di_{bat}/dt ? At $t = 3.0 \mu\text{s}$, what are (c) i_{bat} and (d) di_{bat}/dt ? A long time later, what are (e) i_{bat} and (f) di_{bat}/dt ?

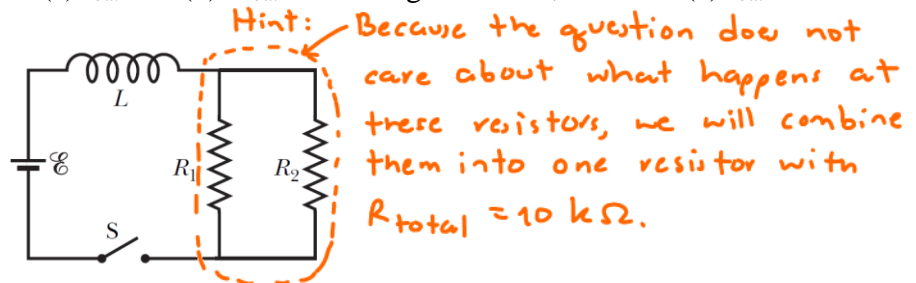


Figure 3: [HRW, 9E, Fig. 30-74]

Ans: (a) 0 ; (b) 800 A/s ; (c) $1.8 \mu\text{A}$; (d) 439 A/s ; (e) 4 mA ; (f) 0

10. [HRW, 9E, P30.95] In Figure 4, $R_1 = 8.0 \Omega$, $R_2 = 10 \Omega$, $L_1 = 0.30 \text{ H}$, $L_2 = 0.20 \text{ H}$, and the ideal battery has $\mathcal{E} = 6.0 \text{ V}$. (a) Just after switch S is closed, at what rate is the current in inductor 1 changing? (b) When the circuit is in the steady state, what is the current in inductor 1?

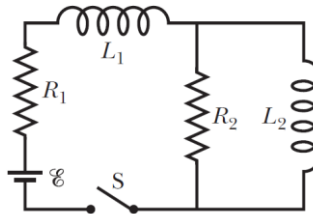
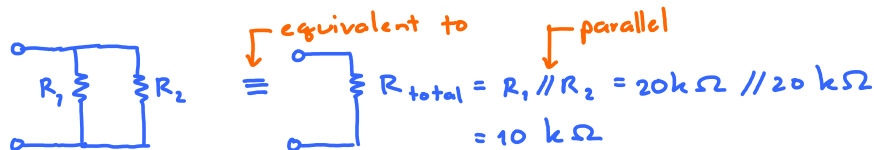


Figure 4: [HRW, 9E, Fig. 30-75]

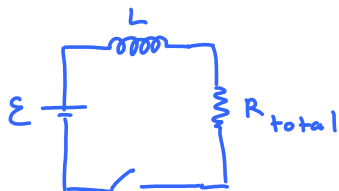
Ans: (a) 20 A/s ; (b) 0.75 A

[HRW, 9E, P30.91]

Observe that the question does not ask anything specifically about quantities involving R_1 and R_2 . So we can simplify the circuit by replacing them with a resistor whose resistance is the total resistance.



So, the overall circuit becomes a simple RL circuit:



The sw is closed at time $t = t_0$ where $t_0 = 0$.

First, we consider the circuit **before** the sw is closed.

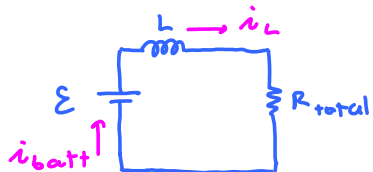
It is given that the sw has been open for a long time, therefore, for $t < t_0$, the inductor acts as an ordinary connecting wire.



Of course, with the broken connection, the current can't flow in this loop. Therefore the current through the inductor is

$$i_L = 0 \text{ A.}$$

Next, we consider the circuit **just after** the sw is closed.



$t = t_0^+ = 0^+$
 Because t_0^+ is so close to t_0 , we know that the value of i_L can not jump to other value.

$$\text{Therefore, } i_L(t_0^+) = 0 \text{ A.}$$

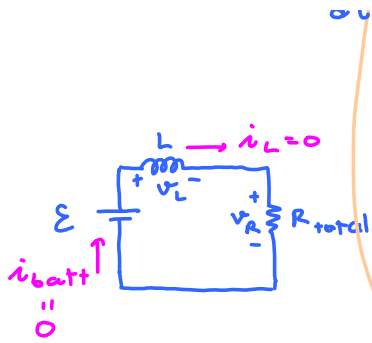
(a) Since the circuit is simply one loop, the current must be the same across all elements. Therefore $i_{batt} = 0 \text{ A}$.

(b) Again, because the same current passes through all circuit elements, we have

$$\frac{di_{batt}}{dt} = \frac{di_L}{dt}$$

We know that $L \frac{di_L}{dt} = v_L$. Therefore, $\frac{di_L}{dt} = \frac{v_L}{L}$.

↑ the voltage across the inductor.



↳ the voltage across the inductor.

By KVL (loop rule), we have $\epsilon - v_L - v_R = 0$.
 Note that $v_R = 0$ because there is no current through the resistor.
 Therefore, $v_L = \epsilon$.

$$\frac{di_{batt}}{dt} = \frac{\epsilon}{L} = \frac{40}{50 \text{ m}} = 0.8 \text{ kA/s} = 800 \text{ A/s}.$$

(c) In fact, we can write down the complete expression for i_L for $t > t_0$.
 In class, we have seen that

$$i_L(t) = e^{a(t-t_0)} (i_0 - i_\infty) + i_\infty,$$

where $a = -\frac{R_{total}}{L}$ and $i_\infty = \frac{\epsilon}{R_{total}}$.

Here, $t_0 = 0$ and $i_0 = i_L(t_0) = 0$

From the discussion before part (a) above, we know that $i_L(t) = 0$ for $t < t_0$. Again, because the value of i_L can not jump, we have $i_L(t_0) = 0$ also.

So,

$$i_L(t) = \frac{\epsilon}{R_{total}} \left(1 - e^{-\frac{R_{total}t}{L}} \right) \text{ for } t \geq 0.$$

$\begin{matrix} 40 \text{ V} \\ \downarrow \\ \epsilon \\ \uparrow \\ 10 \text{ k}\Omega \\ R_{total} \end{matrix}$
 $\begin{matrix} 10 \text{ k}\Omega \\ \downarrow \\ R_{total} \\ \uparrow \\ 50 \text{ mH} \\ L \end{matrix}$

At $t = 3 \mu\text{s}$, we have $i_L(3 \mu\text{s}) \approx 1.8 \text{ mA}$.

(d) By KVL (loop rule), $\epsilon - v_L - v_R = 0$

$$\epsilon - L \frac{di_L}{dt} - i_L R = 0$$

$$\frac{di_L}{dt} = -\frac{R}{L} i_L + \frac{\epsilon}{L} \leftarrow \text{same as what we've seen in class.}$$

$$\approx 439 \text{ A/s}.$$

As in part (b), $\frac{di_{batt}}{dt} = \frac{di_L}{dt} \approx 439 \text{ A/s}$.

In the last part of the question, we look at the circuit again a long time later.
 So, the inductor becomes an ordinary connecting wire.

(e)



Therefore, $i_{batt} = i_L = \frac{\epsilon}{R} = 4 \text{ mA}$.

(e)



Therefore, $i_{\text{batt}} = i_L = \frac{\mathcal{E}}{R} = 4 \text{ mA}$.

$10 \text{ k}\Omega$

(f) As in part (d), $\frac{di_L}{dt} = -\frac{R}{L}i_L + \frac{\mathcal{E}}{L} = 0$.

So, $\frac{di_{\text{batt}}}{dt} = \frac{di_L}{dt} = 0 \text{ A/s}$.