Practice Questions for “Induction and Inductance”

1. [HRW, 9E, P30.40] The inductance of a closely packed coil of 400 turns is 8.0 mH. Calculate the magnetic flux through the coil when the current is 5.0 mA.
   Ans: 10\(^{-7}\) Wb

2. [HRW, 9E, P30.44] A 12 H inductor carries a current of 2.0 A. At what rate must the current be changed to produce a 60 V emf in the inductor?
   Ans: 5 A/s

3. [HRW, 9E, P30.45] At a given instant the current and self-induced emf in an inductor are directed as indicated in Figure 1. (a) Is the current increasing or decreasing? (b) The induced emf is 17 V, and the rate of change of the current is 25 kA/s; find the inductance.
   Ans: (a) decreasing (b) 6.8\( \times 10^{-4}\) H.

4. A conducting loop of area 240 cm\(^2\) and resistance 12\( \Omega\) lies at right angles to a spatially uniform magnetic field. The loop carries an induced current of 320 mA. At what rate is the magnetic field changing?
   Ans: 160 T/s

5. A 60 mA current is flowing in a 100 mH inductor. Over a period of 1.0 ms the current is reversed, going steadily to 60 mA in the opposite direction. What is the inductor emf during this time?
   Ans: 12V

6. The current in a 2.0 H inductor is given by \(i(t) = 3t^2 + 15t + 8\) where \(t\) is in seconds and \(i(t)\) is in amperes. Find an expression for the magnitude of the inductor emf.
   Ans: 12\( t + 30\) V

7. [Modified from HRW, 9E, P30.53] A solenoid having an inductance of 6.30 \(\mu\)H is connected in series with a 1.20 k\(\Omega\) resistor. A 14.0 V battery is connected across the pair as shown in Figure 2. Suppose, at time \(t_0 = 0\) s, the current \(i\) through the resistor is 0 A.
   a. A long time later (\(t \to \infty\)), what is the value of \(i\).
      \(i_\infty = \frac{E}{R} \approx 11.7\) mA
   b. Find the inductive time constant \(\tau_L\).
      \(\tau_L = \frac{L}{R} \approx 5.25\) ns
Additional Practice Questions for “Induction and Inductance”

8. [Modified from HRW, 9E, P30.53] (Continued from Q7) A solenoid having an inductance of 6.30 \( \mu \text{H} \) is connected in series with a 1.20 k\( \Omega \) resistor. A 14.0 V battery is connected across the pair as shown in Figure 2. Suppose, at time \( t_0 = 0 \) s, the current \( i \) through the resistor is 0 A.
   a. Find the current \( i \) at an arbitrary time \( t > 0 \) s. (This will be a function of \( t \).)
   b. Find the current \( i \) at time \( t = 0.000000001 \) s.
   c. Find the time at which the current through the resistor reaches 80.0% of its final value.
   d. What is the current through the resistor at time \( t = t_0 + \tau_L \)?

   Ans: (1) 11.7 mA; (b) 5.25 ns; (c) \( 8.19 \times 10^{-11} \) mA; (d) 2.0 mA; (e) 8.45 ns; (f) 7.38 mA.

9. [HRW, 9E, P30.91] In the circuit of Figure 3, \( R_1 = 20 \) k\( \Omega \), \( R_2 = 20 \) k\( \Omega \), \( L = 50 \) mH, and the ideal battery has \( E = 40 \) V. Switch S has been open for a long time when it is closed at time \( t = 0 \). Just after the switch is closed, what are (a) the current \( i_{\text{bat}} \) through the battery and (b) the rate \( di_{\text{bat}}/dt \)? At \( t = 3.0 \) \( \mu \)s, what are (c) \( i_{\text{bat}} \) and (d) \( di_{\text{bat}}/dt \)? A long time later, what are (e) \( i_{\text{bat}} \) and (f) \( di_{\text{bat}}/dt \)?

   Ans: (a) 0; (b) 800 A/s; (c) 1.8 \( \mu \)A; (d) 439 A/s; (e) 4mA; (f) 0

10. [HRW, 9E, P30.95] In Figure 4, \( R_1 = 8.0 \) \( \Omega \), \( R_2 = 10 \) \( \Omega \), \( L_1 = 0.30 \) H, \( L_2 = 0.20 \) H, and the ideal battery has \( E = 6.0 \) V. (a) Just after switch S is closed, at what rate is the current in inductor 1 changing? (b) When the circuit is in the steady state, what is the current in inductor 1?

   Ans: (a) 20 A/s; (b) 0.75 A
Observe that the question does not ask anything specifically about quantities involving $R_1$ and $R_2$. So, we can simplify the circuit by replacing them with a resistor whose resistance is the total resistance.

\[ R_{\text{total}} = R_1 \parallel R_2 = 20 \, \text{k}\Omega \parallel 20 \, \text{k}\Omega = 10 \, \text{k}\Omega \]

So, the overall circuit becomes a simple RL circuit:

The switch is closed at time $t = t_0$, where $t_0 = 0$.

First, we consider the circuit before the switch is closed.

It is given that the switch has been open for a long time, therefore, for $t < t_0$, the inductor acts as an ordinary connecting wire.

Of course, with the broken connection, the current can’t flow in this loop. Therefore, the current through the inductor is $i_L = 0 \, \text{A}$.

Next, we consider the circuit just after the switch is closed.

At $t = t_0$, $L = 0$

Because $t_0$ is so close to $t_0$, we know that the value of $i_L$ cannot jump to other values.

Therefore, $i_L(t_0) = 0 \, \text{A}$.

(a) Since the circuit is simply one loop, the current must be the same across all elements. Therefore, $i_{\text{batt}} = 0 \, \text{A}$.

(b) Again, because the same current passes through all circuit elements, we have

\[ \frac{di_{\text{batt}}}{dt} = \frac{di_L}{dt} \]

We know that $L \frac{di_L}{dt} = v_L$. Therefore, \[ \frac{di_L}{dt} = \frac{v_L}{L} \]

the voltage across the inductor.
(c) In fact, we can write down the complete expression for $i_L$ for $t > t_0$.

In class, we have seen that

$$i_L(t) = e^{-\frac{t-t_0}{\alpha}} (i_0 - \dot{i}_0) + \dot{i}_0,$$

where $\alpha = \frac{R_{\text{total}}}{L}$ and $\dot{i}_0 = \frac{E}{R_{\text{total}}}$.

Here, $t_0 = 0$ and $\dot{i}_0 = i_L(t_0) = 0$.

From the discussion before part (a) above, we know that $i_L(t) = 0$ for $t < t_0$. Again, because the value of $i_L$ can not jump, we have $i_L(t_0) = 0$ also.

So,

$$i_L(t) = \frac{E}{R_{\text{total}}} \left(1 - e^{-\frac{t}{\alpha}}\right) \quad \text{for } t > 0.$$  

At $t = 3 \mu$s, we have $i_L(3 \mu$s) $\approx 1.8$ mA.

(d) By KVL (loop rule),

$$E - V_L - V_R = 0$$

$$E - L \frac{d\dot{i}_L}{dt} - i_L R = 0$$

$$\frac{d\dot{i}_L}{dt} = -\frac{R}{L} \dot{i}_L + \frac{E}{L}$$

$\approx 439$ A/s.

As in part (b),

$$\frac{d\dot{i}_{\text{batt}}}{dt} = \frac{d\dot{i}_L}{dt} = 439 \text{ A/s}.$$

In the last part of the question, we look at the circuit again a long time later. So, the inductor becomes an ordinary connecting wire.

(e) Therefore, $\dot{i}_{\text{batt}} = \dot{i}_L = \frac{E}{R} = 4$ mA.
(e) Therefore, \( i_{\text{batt}} = \frac{\varepsilon}{R} = 4 \text{ mA}. \)

(f) As in part (d), \( \frac{d\dot{i}_L}{dt} = -\frac{R}{L} \dot{i}_L + \frac{\varepsilon}{L} = 0. \) So, \( \frac{d\dot{i}_{\text{batt}}}{dt} = \frac{d\dot{i}_L}{dt} = 0 \text{ A/s}. \)