Sirindhorn International Institute of Technology
Thammasat University
Department of Common and Graduate Studies

MAS 116: Solution for Problem Set 6 (Part 1)

Semester/Year: 3/2008

Course Title: MAS116 (Mathematics I)

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Textbook: Howard Anton, Irl Bivens, and Stephen Davis, *Calculus*, 8th Edition, Wiley (2005).

Due date: Not Due

Section 5.2: 9, 38
Section 5.3: 2, 8, 22
Section 5.5: 15, 24, 26, 27, 30
Section 5.6: 8, 12, 42, 44, 45, 64
Section 8.2: 2, 10, 24, 30, 44
Section 8.2: 2, 3, 4, 12, 14, 42

Remarks:

1. Only ten of the above problems will be graded. Of course, you do not know which problems will be selected; so you should work on all of them.
2. Late submission will not be accepted.
3. Write down all the steps that you have done to obtain your answers. You will not get full credit even when your answer is correct without showing how you get your answer.

Please submit all homework in the MAS116 box in front of the CGS Department which is located on the 3rd floor of RS building.

Wednesday, May 13, 2009
12:15 PM

Exercise Set 5.2

(9) (c)

$$\int x^3\sqrt{x}\,dx = \int x^3 x^{\frac{1}{2}}\,dx = \int x^{\frac{7}{2}}\,dx$$

$$= \frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1} = \boxed{\frac{2}{9}x^{\frac{9}{2}} + C}$$

(38) (a) $\frac{dy}{dx} = \frac{1}{(2x)^3} = \frac{1}{8x^3} = \frac{1}{8}x^{-3}$

$$y = \int \frac{1}{8}x^{-3}\,dx = \frac{1}{8}\frac{x^{-3+1}}{(-3+1)} + C = -\frac{1}{16}x^{-2} + C$$

$$= -\frac{1}{16x^2} + C$$

$$y(1) = -\frac{1}{16} + C = 0$$

So, $c = \frac{1}{16}$

Therefore $\boxed{y(x) = -\frac{1}{16x^2} + \frac{1}{16}}$

Exercise Set 5.3

(2) (b) $\int y\sqrt{1+2y^2}\,dy$

$u = 1+2y^2$

$\frac{du}{dy} = 4y$

$\frac{1}{4}du = y\,dy$

$$= \int \sqrt{u}\,\frac{1}{4}\,du$$

$$= \frac{1}{4}\int u^{\frac{1}{2}}\,du$$

$$= \frac{1}{4}\frac{u^{3/2}}{3/2} + C$$

$$= \boxed{\frac{1}{6}(1+2y^2)^{3/2} + C}$$

(8) $\int x^3\sqrt{5+x^4}\,dx = \int \frac{1}{4}\sqrt{u}\,du = \frac{1}{4}\frac{u^{3/2}}{3/2} + C$

$u = 5 + x^4$

$\frac{du}{dx} = 4x^3$

$= \boxed{\frac{1}{6}\left(5 + x^4\right)^{3/2} + C}$

$\frac{1}{4} du = x^3 dx$

(22) $\int \cos 2t \sin^5 2t \, dt = \int u^5 \frac{1}{2} du = \frac{1}{2} \frac{u^6}{6} + C$

$u = \sin 2t$

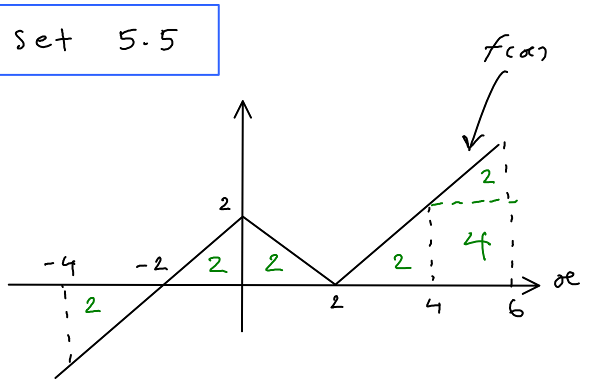
$= \boxed{\frac{1}{12} \sin^6 2t + C}$

$\frac{du}{dt} = (\cos 2t) 2$

$\frac{1}{2} du = (\cos 2t) dt$

Exercise set 5.5

(15)



In this problem, we use the area formulas from geometry to evaluate the integrals.

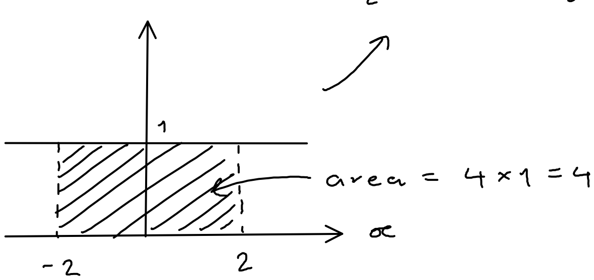
(a) $\frac{1}{2} \times 2 \times 2 = 2$

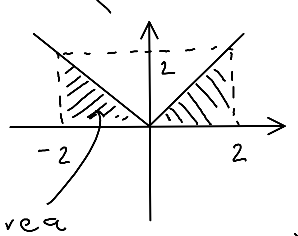
(b) $2 \times 2 = 4$

(c) $2 + 2 + 2 + 4 = 10$

(d) $\left((-2) + 2\right) + 10 = 10$

(24) $\int_{-2}^{2} 1 - 3|x| \, dx = \int_{-2}^{2} 1 \, dx - 3\int_{-2}^{2} |x| \, dx = 4 - 3 \times 4 = \boxed{-8}$





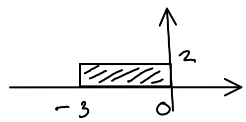
area

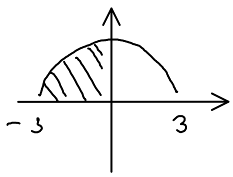
$= \cancel{2} \times \left(\frac{1}{\cancel{2}} \times 2 \times 2\right)$

$$= 4$$

(26) $$\int_{-3}^{0} 2 + \sqrt{9 - x^2}\, dx = \int_{-3}^{0} 2\, dx + \int_{-3}^{0} \sqrt{9 - x^2}\, dx$$

$$= (2 \times 3) + \frac{1}{4}\pi \times 3^2$$





$$= \boxed{6 + \frac{9}{4}\pi}$$

(27) (a) For $2 \leq x \leq 3$,

$$-3 \leq -x \leq -2$$

$$-2 \leq 1 - x \leq -1$$

Therefore, $\dfrac{\sqrt{x}}{1-x}$ is negative on $[2,3]$

Hence, $\displaystyle\int_{2}^{3} \frac{\sqrt{x}}{1-x}\, dx$ is also $\boxed{\text{negative}}$

(b) For all x, $-1 \leq \cos x \leq 1$

So, $3 - \cos x$ is positive

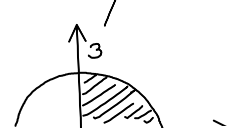
Hence, $\dfrac{x^2}{3 - \cos x}$ is also positive.

Therefore, $\displaystyle\int_{0}^{4} \frac{x^2}{3 - \cos x}\, dx$ is $\boxed{\text{positive}}$

(30) $$\int_{0}^{3} \sqrt{6x - x^2}\, dx = \int_{0}^{3} \sqrt{3^2 - (x-3)^2}\, dx = \frac{1}{4}(\pi \times 3^2) = \boxed{\frac{9\pi}{4}}$$

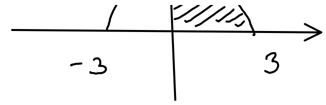
$$6x - x^2 = -(x^2 - 6x)$$

$$= -(x^2 - 3 \times 2x + 3^2 - 3^2)$$



$$= -\left((x-3)^2 - 3^2\right)$$

$$= 3^2 - (x-3)^2$$



Exercise Set 5.6

8. $$\int_{-1}^{2} 4x(1-x^2)\,dx = \int_{-1}^{2} 4x - 4x^3\,dx = 4\frac{x^2}{2} - 4\frac{x^4}{4}\Big]_{-1}^{2}$$

$$= 2x^2 - x^4\Big]_{-1}^{2} = (2\times 4 - 16) - (2\times 1 - 1)$$

$$= (8-16) - (2-1) = -8-1 = \boxed{-9}$$

12. $$\int_{1}^{4} \frac{1}{x\sqrt{x}}\,dx = \int_{1}^{4} \frac{1}{x\,x^{1/2}}\,dx = \int_{1}^{4} \frac{1}{x^{3/2}}\,dx = \int_{1}^{4} x^{-3/2}\,dx$$

$$= \frac{x^{-\frac{1}{2}}}{\left(-\frac{1}{2}\right)}\Big]_{1}^{4} = \frac{-2}{\sqrt{x}}\Big]_{1}^{4} = -2\left(\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{1}}\right)$$

$$= -2\ \frac{1}{2} - 1) = (-2)\left(-\frac{1}{2}\right) = \boxed{1}$$

42. $$F(x) = \int_{\pi/4}^{x} \cos 2t\,dt$$

(a)

$$F'(x) = \boxed{\cos 2x}$$

(b)

$$F(x) = \int_{\pi/2}^{2x} \frac{1}{2}\cos u\,du = \frac{1}{2}\sin u\Big]_{\pi/2}^{2x}$$

$u = 2t$

$du = 2dt$

$dt = \frac{1}{2}du$

$$= \frac{1}{2}\left(\sin 2x - \sin\frac{\pi}{2}\right)$$

1

$$= \frac{1}{2}\sin 2x - \frac{1}{2}$$

$$F'(x) = \frac{1}{2}\cos(2x)\frac{d}{dx}(2x) - 0$$

$$= \frac{1}{\cancel{2}}\cos(2x) \times \cancel{2}$$

$$= \cos(2x)$$

$$= \cos(2x)$$

44 (a) $\boxed{\dfrac{1}{1+\sqrt{x}}}$

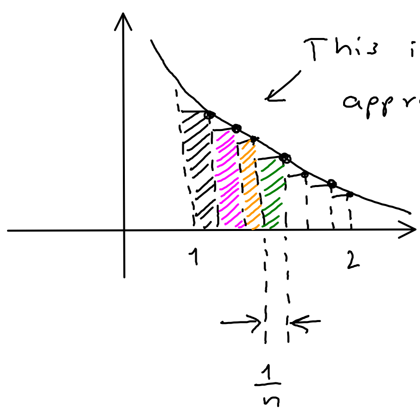
(b) $\boxed{\dfrac{1}{x^2+3x-4}}$

45 $$\frac{d}{dx}\int_x^0 t\sec t\,dt = \frac{d}{dx}\left(-\int_0^x t\sec t\,dt\right)$$

$$= -\frac{d}{dx}\int_0^x t\sec t\,dt$$

$$= \boxed{-x\sec x} = \frac{-x}{\cos x}$$

64 $$\frac{n}{(n+k)^2} = \frac{1}{n}\,\frac{n^2}{(n+k)^2} = \frac{1}{n}\,\frac{1}{\left(1+\frac{k}{n}\right)^2}$$



This is the right endpoint approximation of

$$\int_1^2 \frac{1}{x^2}\,dx = -\frac{1}{x}\Big]_1^2 = -\left(\frac{1}{2}-1\right)$$
$$= \frac{1}{2}$$

So, $\displaystyle\lim_{n\to\infty}\sum_{n=1}^{n}\frac{n}{(n+k)^2} = \boxed{\frac{1}{2}}$

Remark: If the interval is $[0,1]$, then the corresponding integration is

$$\int_0^1 \frac{1}{(1+x)^2}\,dx$$

which we can evaluate by u-substitution method

$$\int_0^1 \frac{1}{(1+x)^2}\,dx \underset{u=1+x}{=} \int_1^2 \frac{1}{u^2}\,du.$$ ← Same as above.

Exercise Set 8.2

(2) $\int x e^{3x}\,dx = \boxed{\frac{1}{3}x e^{3x} - \frac{1}{9}e^{3x} + C}$

$$\begin{array}{ccc} x & \overset{+}{\searrow} & e^{3x} \\ 1 & \overset{-}{\searrow} & \frac{1}{3}e^{3x} \\ 0 & \xrightarrow[+]{} & \frac{1}{9}e^{3x} \end{array}$$

(10) $\int \sqrt{x}\ln x\,dx = \frac{2}{3}x^{3/2}\ln x - \underbrace{\int \frac{1}{x}\times\frac{2}{3}x^{3/2}\,dx}$

$$\begin{array}{ccc} \ln x & \overset{+}{\searrow} & x^{\frac{1}{2}} \\ \frac{1}{x} & \xrightarrow[-]{} & \frac{2}{3}x^{\frac{3}{2}} \end{array}$$

$= \frac{2}{3}\int x^{1/2}\,dx$

$= \frac{2}{3}\,\frac{2}{3}x^{3/2} + C$

$\int \sqrt{x}\ln x\,dx = \boxed{\frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C}$

(24) $\int \cos(\ln x)\,dx = x\cos\ln x + \underbrace{\int \sin(\ln x)\,dx}$

$$\begin{array}{ccc} \cos(\ln x) & \overset{+}{\searrow} & 1 \\ -\sin(\ln x)\frac{1}{x} & \xrightarrow[-]{} & x \end{array}$$

$$\begin{array}{ccc} \sin(\ln x) & \overset{+}{\searrow} & 1 \\ \cos(\ln x)\frac{1}{x} & - & x \end{array}$$

$= x\sin(\ln x) - \int \cos(\ln x)\,dx$

So, $\int \cos(\ln x)\,dx = x\cos(\ln x) + x\sin(\ln x) - \int\cos(\ln x)\,dx$

$2\int\cos(\ln x)\,dx = x(\cos\ln x + \sin\ln x)$

$\cos(\ln x) = \boxed{\frac{x}{2}\left(\cos(\ln x) + \sin(\ln x)\right) + C}$

(44) $\int (x^2+x+1)\sin x\,dx = -(x^2+x+1)\cos x + (2x+1)\sin x + 2\cos x + C$

$$\begin{array}{ccc} x^2+x+1 & \overset{+}{\searrow} & \sin x \\ 2x+1 & \overset{-}{\searrow} & -\cos x \\ 2 & \overset{+}{\searrow} & -\sin x \\ 0 & & \cos x \end{array}$$

$= \boxed{-(x^2+x+1)\cos x + (2x+1)\sin x + C}$

(30) $\int_0^1 x e^{-ax}\,dx = -\frac{1}{a}x e^{-ax} - \frac{1}{a^2}e^{-ax}\Big]_0^1$

$= \left(-\frac{1}{a}e^{-a} - \frac{1}{a^2}e^{-a}\right) - \left(0 - \frac{1}{a^2}\right)$

$= \underline{-\left(\frac{a+1}{a^2}\right)e^{-a} + \frac{1}{a^2}}$

$$\begin{array}{ccc} x & \overset{+}{\searrow} & e^{-ax} \\ 1 & \overset{-}{\searrow} & -\frac{1}{a}e^{-ax} \end{array}$$

$\boxed{0 \quad + \frac{1}{a^2}e^{-ax}} = \boxed{-\frac{6}{25}e^{-5} + \frac{1}{25}}$

when $a = 5$

Exercise Set 8.8

(2) The limits of integration are all finite.
So, we look for infinite discontinuities which usually happen when denominator $\to 0$.

(a) $\frac{1}{x^p} \leftarrow$ This gives infinite discontinuities when $x = 0$. However, p must be $\boxed{>0}$ to keep it in the denominator.

(b) $\frac{1}{x-p} \leftarrow$ This function is 0 at $x = p$. To include this point in the integration, we need $\boxed{1 \le p \le 2}$

(c) $e^{-px} \leftarrow$ This function is bounded on bounded interval. Therefore, no value of p will make this function improper.

(3) $$\int_0^b e^{-2x}\,dx = -\frac{1}{2}e^{-2x}\Big]_0^b = -\frac{1}{2}e^{-2b} - \left(-\frac{1}{2}\right) \xrightarrow{b\to\infty} \boxed{\frac{1}{2}}$$

0 as $b \to \infty$

(4) $$\int_{-1}^{b} \frac{x}{1+x^2}\,dx = \frac{1}{2}\int_2^{1+b^2} \frac{1}{u}\,du = \frac{1}{2}\ln|u|\Big]_2^{1+b^2} = \frac{1}{2}\ln\frac{1+b^2}{2} \to \infty$$ as $b \to \infty$

So, $\boxed{\text{divergent}}$

$u = 1 + x^2$

$du = 2x\,dx$

$x\,dx = \frac{1}{2}du$

(12) $$\int_a^0 \frac{e^x\,dx}{3-2e^x} = -\frac{1}{2}\int_{3-2e^a}^{1} \frac{1}{u}\,du = -\frac{1}{2}\ln|u|\Big]_{3-2e^a}^{1}$$

$u = 3 - 2e^x$

$du = -2e^x\,dx$

$e^x\,dx = -\frac{1}{2}du$

$$= -\frac{1}{2}\left(\ln\frac{1}{3-2e^a}\right)$$

as $a \to -\infty$

$$= -\frac{1}{2}\ln\frac{1}{3}$$

$$= \boxed{\frac{1}{2}\ln 3}$$

(14) $$\int_a^b \frac{x}{\sqrt{x^2+2}}\,dx = \frac{1}{2}\int_{a^2+2}^{b^2+2} \frac{1}{\sqrt{u}}\,du = \sqrt{u}\Big]_{a^2+2}^{b^2+2} = \sqrt{b^2+2} - \sqrt{a^2+2}$$

$u = x^2 + 2$

$$du = 2x\,dx$$

$$x\,dx = \frac{1}{2}du$$

$$\int_0^b \frac{x}{\sqrt{x^2+2}}\,dx = \sqrt{b^2+2} - \sqrt{2} \longrightarrow \infty \quad \text{as} \quad b \to \infty$$

Because $\int_0^\infty \frac{x}{\sqrt{x^2+2}}\,dx$ diverges, we conclude that

$$\int_{-\infty}^{\infty} \frac{x}{\sqrt{x^2+2}}\,dx \quad \boxed{\text{diverges}}$$

(by definition)

42 $$\int \frac{\ln x}{x^2}\,dx = -\frac{1}{x}\ln x + \int \frac{1}{x^2}\,dx$$

$$= -\frac{1}{x}\ln x - \frac{1}{x} + C$$

$$\begin{array}{cc} \ln x & \frac{1}{x^2} \\ & + \\ \frac{1}{x} & \xrightarrow{-} \; -\frac{1}{x} \end{array}$$

$$\int_1^b \frac{\ln x}{x^2}\,dx = -\frac{1}{x}\ln x - \frac{1}{x}\Big]_1^b = \left(-\frac{\ln b}{b} - \frac{1}{b}\right) - (0 - 1)$$

$$= -\left(\frac{\ln b}{b} - \frac{1}{b} - 1\right)$$

Note that $\lim_{b \to \infty} \frac{\ln b}{b} = \lim_{b \to \infty} \frac{1/b}{1} = 0$

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$$\lim_{b \to \infty} \int_1^b \frac{\ln x}{x^2}\,dx = -(0 - 0 - 1) = \boxed{1}$$