Sirindhorn International Institute of Technology
Thammasat University
Department of Common and Graduate Studies

MAS 116: Solution for Problem Set 5

Semester/Year: 3/2008

Course Title: MAS116 (Mathematics I)

Instructor: Dr. Prapun Suksompong (prapun@siit.tu.ac.th)

Textbook: Howard Anton, Irl Bivens, and Stephen Davis, *Calculus*, 8th Edition, Wiley (2005).

Due date: May 15 (Friday)

Section 4.1: 1, 2, 6, 12, 18
Section 4.2: 8, 16, 24
Section 4.4: 6, 10, 48a
Section 4.5: 2

Remarks:

1. Only ten of the above problems will be graded. Of course, you do not know which problems will be selected; so you should work on all of them.
2. Late submission will not be accepted.
3. Write down all the steps that you have done to obtain your answers. You will not get full credit even when your answer is correct without showing how you get your answer.

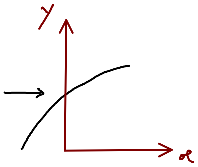
Please submit all homework in the MAS116 box in front of the CGS Department which is located on the 3rd floor of RS building.

Tuesday, May 05, 2009
1:22 PM

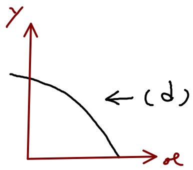
Exercise Set 4.1

Note that "increasing" here means "strictly increasing" and "decreasing" here means "strictly decreasing".

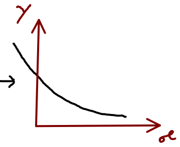
① (b) $f' > 0$, $f'' < 0$



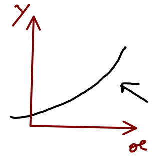
(d) $f' < 0$, $f'' < 0$



(c) $f' < 0$, $f'' > 0$

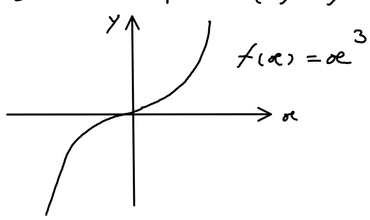


(a) $f' > 0$, $f'' > 0$

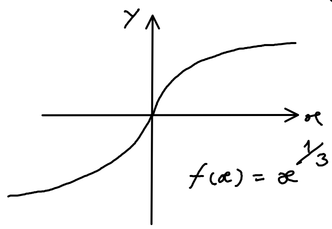


② All of these functions have inflection point @ origin.

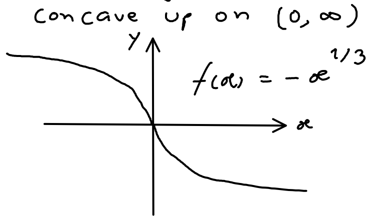
(a) increasing on $\mathbb{R}$
concave up on $(0, \infty)$



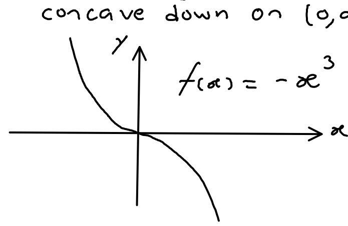
(b) increasing on $\mathbb{R}$
concave down on $(0, \infty)$



(b) decreasing on $\mathbb{R}$
concave up on $(0, \infty)$



(d) decreasing on $\mathbb{R}$
concave down on $(0, \infty)$



⑥ In this problem, you are given a graph of $y = f'(x)$.
First, note that
f' exists everywhere.
Hence, f is
continuous everywhere.

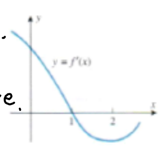


Figure Ex-6

Note that $f'(1) = 0$ is not a problem here

(a) Because $f'(x) > 0$ on $(0, 1)$,
f is increasing on $[0, 1]$.
Therefore, $\boxed{f(0) < f(1).}$

(b) Because $f'(x) < 0$ on $(1, 2)$,
f is increasing on $[1, 2]$.
Therefore, $\boxed{f(1) < f(2)}$

(c) $\boxed{f'(0) > 0}$ by inspection.

(c) $\boxed{f'(0) > 0}$ by inspection.

(d) $\boxed{f'(1) = 0}$ by inspection.

(e) $f'(x)$ is decreasing at $x = 0$.
Therefore, $\boxed{f''(0) < 0}$

(f) $f''(2) = 0$ because f' has a minimum there.

(12) $f(x) = 5 - 4x - x^2$

$x = (-2)$

$f'(x) = -4 - 2x = 0$

	sign of $f'(x)$
$x < -2$	+
$x = -2$	
$x > -2$	−

(a) f is increasing on $\boxed{(-\infty, -2]}$

(b) f is decreasing on $\boxed{[-2, +\infty)}$

$f''(x) = -2 < 0$

(c) $\boxed{\text{nowhere}}$

(d) $\boxed{(-\infty, \infty)}$

(e) $\boxed{\text{none}}$

(18) $f(x) = \dfrac{x}{x^2 + 2}$

$$f'(x) = \frac{(x^2+2) \times 1 - x(2x)}{(x^2+2)^2}$$

$$= \frac{x^2 + 2 - 2x^2}{(x^2+2)^2}$$

$$= \frac{2 - x^2}{(x^2+2)^2}$$

$f'(x) = 0$ when $2 - x^2 = 0$
or
$x = \pm\sqrt{2}$

	Sign of $f'(x)$
$x < -\sqrt{2}$	−
$-\sqrt{2}$	
$-\sqrt{2} < x < \sqrt{2}$	+
$+\sqrt{2}$	
$x > \sqrt{2}$	−

$$f''(x) = \frac{(x^2+2)^{\cancel{4}\ 2}(-2x) - (2-x^2)\,2\,\cancel{(x^2+2)}(2x)}{(x^2+2)^{\cancel{4}\ 3}}$$

$$= \frac{(2x)\left(-x^2 - 2 - 4 + 2x^2\right)}{(x^2+2)^3}$$

$$= \frac{(2x)(x^2-6)}{(x^2+2)^3}$$

$f''(x) = 0$ when $x = 0$
or
$x^2 - 6 = 0$ or $x = \pm\sqrt{6}$

	sign of $(2x)$	(x^2-6)	$f''(x)$
$x < -\sqrt{6}$	−	+	−
$x = -\sqrt{6}$			
$-\sqrt{6} < x < 0$	−	−	+
$x = 0$			
$0 < x < \sqrt{6}$	+	−	−
$x = +\sqrt{6}$			

$$f'(x) = \frac{(x^2+2)\times 1 - x(2x)}{(x^2+2)^2}$$

$$= \frac{x^2+2-2x^2}{(x^2+2)^2}$$

$$= \frac{2-x^2}{(x^2+2)^2}$$

$f'(x) = 0$ when $2-x^2=0$
or
$x = \pm\sqrt{2}$

	Sign of $f'(x)$
$x<-\sqrt{2}$	$-$
$-\sqrt{2}$	
$-\sqrt{2}<x<\sqrt{2}$	$+$
$+\sqrt{2}$	
$x>\sqrt{2}$	$-$

(a) f is increasing on

$\boxed{[-\sqrt{2}, \sqrt{2}]}$

(b) f is decreasing on

$\boxed{(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)}$

$$f''(x) = \frac{(x^2+2)^{\cancel{2}}(-2x) - (2-x^2)2\cancel{(x^2+2)}(2x)}{(x^2+2)^{\cancel{4}\,3}}$$

$$= \frac{(2x)(-x^2-2-4+2x^2)}{(x^2+2)^3}$$

$$= \frac{(2x)(x^2-6)}{(x^2+2)^3}$$

$f''(x) = 0$ when $x=0$
or
$x^2-6=0$ or $x=\pm\sqrt{6}$

	sign of $(2x)$	(x^2-6)	$f''(x)$
$x<-\sqrt{6}$	$-$	$+$	$-$
$x=-\sqrt{6}$			
$-\sqrt{6}<x<0$	$-$	$-$	$+$
$x=0$			
$0<x<\sqrt{6}$	$+$	$-$	$-$
$x=+\sqrt{6}$			
$x>\sqrt{6}$	$+$	$+$	$+$

(c) f is concave up on

$\boxed{(-\sqrt{6}, 0) \text{ and } (\sqrt{6}, \infty)}$

(d) f is concave down on

$\boxed{(-\infty, -\sqrt{6}) \text{ and } (0, \sqrt{6})}$

(e) $x = -\sqrt{6}, 0, +\sqrt{6}$

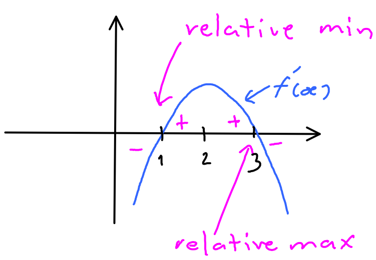
Exercise Set 4.2

8. $f(x) = 3x^4+12x$

$f'(x) = 12x^3+12 = 0 \Rightarrow x=-1$

$\boxed{x=-1}$ is the stationary point.

16.



(a) Relative min @ $\boxed{x=1}$ because f' changes sign from $-$ to $+$ there.

(b) Relative max @ $\boxed{x=3}$ because

f' changes sign from $+$ to $-$ there.

(c) f' is increasing for $x < 2 \Rightarrow f$ concave up

f' is decreasing for $x > 2 \Rightarrow f$ concave down

So, f changes concavity at $\boxed{x = 2.}$

(24) $f(x) = x^4 - 12x^3$

$f'(x) = 4x^3 - 36x^2 = 4x^2(x-9) \Rightarrow f'(x) = 0$ when $x = 0, 9$

$f''(x) = 12x^2 - 36 \times 2x = 12x(x-6) \Rightarrow f''(x) = 0$ when $x = 0, 6$

(a) First derivative test

	sign of x^2	$x-9$	$f'(x)$
$x < 0$	$+$	$-$	$-$
$x = 0$			
$0 < x < 9$	$+$	$-$	$-$
$x = 9$			
$x > 9$	$+$	$+$	$+$

f' changes from "$-$" to "$+$" @ $x = 9$

So $\boxed{\text{relative min at } x = 9.}$

$f(9) = 9^4 - 12 \times 9^3 = \boxed{-2187}$

(b) Second derivative test

$f'(x) = 0$ at $x = 0, 9$

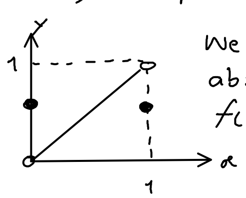
$f''(0) = 0 \Rightarrow$ inconclusive.

$f''(9) = 12 \times 9 \times 3 > 0 \Rightarrow \boxed{\text{Relative min at } x = 9.}$

Exercise Set 4.4

(6) f is strictly increasing on $(0,1)$, one might expect a min at $x = 0$ and a max at $x = 1$.

However, both points are discontinuities of f.



We can't say that f has abs. max at $x = 1$ because $f(1) = \frac{1}{2}$ is less than $f(0.9) = 0.9$.

It seems that we need to pick a point closest to 1 but not exactly 1. This is not achievable. For any point less than 1, we can find another point which is greater than it, but still less than 1.

Similar reason also applies at $x = 0$.

(10) $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12$

$f'(x) = 0$ when $6x^2 + 6x - 12 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = 1, -2$

The function is differentiable everywhere. Therefore, all critical points occur at x where $f'(x) = 0$.

x	$f(x)$	
-3	9	
-2	20	$\leftarrow$ abs. max
1	-7	$\leftarrow$ abs. min
2	4	

$\boxed{\text{The min is } -7 \text{ at } x = 1 \text{ and the max is } 20 \text{ at } x = -2}$

(48) (a) The height corresponds to the "y"

$y = 3 + \sin(2t) - 2\sin^2 t$

$\frac{dy}{dt} = 2\cos(2t) - 2 \times 2 \sin t \cos t$

$= 2\cos(2t) - 2\sin 2t$

$\frac{dy}{dt} = 0$ when $\tan 2t = 1$

$2t = \frac{\pi}{4} + 2n\pi, \frac{5\pi}{4} + 2n\pi$

$t = \frac{\pi}{8} + n\pi, \frac{5\pi}{8} + n\pi$

$= \frac{\pi}{8}, \frac{9\pi}{8}, \frac{5\pi}{8}, \frac{13\pi}{8}$ $\leftarrow$ pick only t that are in $[0, 2\pi]$

t	y	
0	3	
$\pi/8$	$2+\sqrt{2} \approx 3.414$	$\Rightarrow$ max
$5\pi/8$	$2-\sqrt{2} \approx 0.586$	$\Rightarrow$ min
$9\pi/8$	$2+\sqrt{2} \approx 3.414$	$\Rightarrow$ max
$13\pi/8$	$2-\sqrt{2} \approx 0.586$	$\Rightarrow$ min
2π	3	

The height range is $\boxed{[0.586, 3.414]}$

Exercise Set 4.5

(2) Let x and y be the two nonnegative numbers.

② Let x and y be the two nonnegative numbers.

Need $x+y=1$.

To have $y=1-x \geqslant 0$, we need $x \leq 1$.

So, we only consider $0 \leq x \leq 1$.

Want to maximize/minimize x^2+y^2

Note that $x^2+y^2 = x^2+(1-x)^2$

Let

$f(x) = x^2+(1-x)^2 = x^2+1-2x+x^2$

$= 2x^2-2x+1$

$f'(x) = 4x-2 \Rightarrow f'(x)=0$ when $x=\frac{1}{2}$

x	$f(x)$	
0	1	← max
1/2	$2\times\frac{1}{4} - 2\times\frac{1}{2}+1 = \frac{1}{2}-1+1=\frac{1}{2}$	← min
1	1	← max

(a) $f(x)$ is as large as possible when one number is 0 and the other is 1.

(b) $f(x)$ is as small as possible when both number is $\frac{1}{2}$.