Sirindhorn International Institute of Technology
Thammasat University
Department of Common and Graduate Studies

MAS 116: Solution for Problem Set 4

Semester/Year: 3/2008

Course Title: MAS116 (Mathematics I)

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Textbook: Howard Anton, Irl Bivens, and Stephen Davis, *Calculus*, 8^{th} Edition, Wiley (2005).

Due date: May 7 (Thursday)

Section 3.5: 2, 12, 16, 20, 34, 44
Section 3.6: 4, 8, 14, 64, 76
Section 3.7: 10
Section 7.2: 4, 6, 19, 20, 32
Section 7.3: 27
Section 7.5: 6, 10, 26, 38, 48a, 50

Remarks:

1. Only ten of the above problems will be graded. Of course, you do not know which problems will be selected; so you should work on all of them.
2. Late submission will not be accepted.
3. Write down all the steps that you have done to obtain your answers. You will not get full credit even when your answer is correct without showing how you get your answer.

Please submit all homework in the MAS116 box in front of the CGS Department which is located on the 3rd floor of RS building.

Wednesday, April 22, 2009
11:03 AM

Exercise Set 3.5

(2) $f(x) = \frac{5}{x^2} + \sin x$

$f'(x) = 5(-2)x^{-3} + \cos x$

$= \boxed{\frac{-10}{x^3} + \cos x}$

(12) $f(x) = \csc x \cot x$

product rule

$f'(x) = (-\csc x \cot x)\cot x + \csc x(-\csc^2 x)$

$= \boxed{-\csc x \cot^2 x - \csc^3 x}$

(16) $f(x) = \sec^2 x - \tan^2 x = 1$

identity

$f'(x) = \boxed{0}$

(20) $y = \csc x$

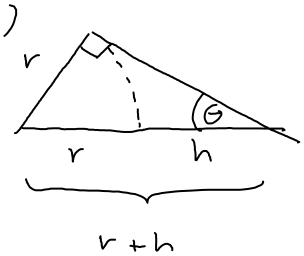
$\frac{dy}{dx} = -\csc x \cot x$

product rule

$\frac{d^2y}{dx^2} = -(-\csc x \cot x)\cot x - \csc x(-\csc^2 x)$

$= \boxed{\csc x \cot^2 x + \csc^3 x}$

(34) (a)



We know that

$\sin\theta = \frac{r}{r+h}$.

$r\sin\theta + h\sin\theta = r$

$h = \frac{r - r\sin\theta}{}$

$$\overline{\sin\theta}$$

$$= \frac{r}{\sin\theta} - r$$

$$= r(\csc\theta - 1)$$

(b) $$\frac{dh}{d\theta} = r\left(-\csc x \cot x\right)$$

$$\left.\frac{dh}{d\theta}\right|_{\theta=30^\circ} = (6378) \times \left(-2 \times \sqrt{3}\right) \quad \text{km/rad}$$

$$= -6378 \times 2 \times \sqrt{3} \times \frac{2\pi}{360} \quad \text{km/degree}$$

$$\approx \boxed{386 \text{ km/degree}}$$

44 $$\lim_{x\to 0} \frac{\tan(x+y) - \tan y}{x} = \lim_{h\to 0} \frac{\tan(y+h) - \tan y}{h}$$

$$= \left.\frac{d}{dx}\tan x\right|_{x=y}$$

$$= \boxed{\sec^2 y}$$

Exercise Set 3.6

4 $f(x) = 5\sqrt{x}$ $\qquad$ $g(x) = 4 + \cos x$

$f'(x) = \dfrac{5}{2\sqrt{x}}$ $\qquad$ $g'(x) = -\sin x$

(a) $$(f\circ g)(x) = f(g(x)) = \boxed{5\sqrt{4+\cos x}}$$

$$(f\circ g)'(x) = f'(g(x))\, g'(x)$$

$$= \frac{5}{2\sqrt{4+\cos x}}(-\sin x) = \boxed{-\frac{5\sin x}{2\sqrt{4+\cos x}}}$$

(b) $$(g\circ f)(x) = g(f(x)) = \boxed{4 + \cos(5\sqrt{x})}$$

(b) $(g\circ f)(x) = g(f(x)) = \boxed{4+\cos(5\sqrt{x})}$

$(g\circ f)'(x) = g'(f(x))\,f'(x)$

$= \boxed{-\sin(5\sqrt{x})\,\frac{5}{2\sqrt{x}}}$

8 $f(x) = (3x^2+2x-1)^6$

$f'(x) = 6(3x^2+2x-1)^5\,(6x+2)$

$= \boxed{12(3x^2+2x-1)^5\,(3x+1)}$

14 $f(x) = \sqrt{\sqrt{x}}$

$f'(x) = \frac{1}{2\sqrt{\sqrt{x}}}\,\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt[4]{x}}\,\frac{1}{2\sqrt{x}} = \boxed{\frac{1}{4x^{3/4}}}$

64 $\frac{d}{dx}\left[f(2\sin x)\right] = f'(2\sin x)\left[\frac{d}{dx}2\sin x\right]$

$= f'(2\sin x)\;2\cos x$

Evaluate at $x = \frac{\pi}{6}$

$\sin x = \frac{1}{2} \qquad \cos x = \frac{\sqrt{3}}{2}$

$f'(2\sin x) = f'(1) = -\frac{5}{2}$ ← This is the slope at $x=1$ on the given graph.

So, $\left.\frac{d}{dx}\left[f(2\sin x)\right]\right|_{x=\frac{\pi}{6}} = -\frac{5}{\cancel{2}} \times \cancel{2} \times \frac{\sqrt{3}}{2}$

$$= \boxed{-\frac{5\sqrt{3}}{2}}$$

(76) (a) f is even

$f(x) = f(-x)$

Differentiate both sides of the eqn. and apply the chain rule.

$f'(x) = f'(-x)(-1) = -f'(-x)$

$f'(-x) = -f'(x)$

So, $f'(x)$ is odd.

(b) f is odd

$f(-x) = -f(x)$

$f'(-x)(-1) = -f'(x)$

Differentiate both sides of the eqn. and apply the chain rule.

$f'(-x) = f'(x)$

So, $f'(x)$ is even.

Exercise Set 3.7

(10) (a) $\sqrt{y} - \sin x = 2$

Note that $\sqrt{y} = 2 + \sin x$

$$\downarrow \frac{d}{dx}$$

$$\frac{1}{2\sqrt{y}}\frac{dy}{dx} - \cos x = 0$$

$$\frac{dy}{dx} = 2\sqrt{y}\cos x$$

(b) $\sqrt{y} = 2 + \sin x$

$$y = (2+\sin x)^2$$

$$\frac{dy}{dx} = 2(2+\sin x)\cos x = \boxed{4\cos x + \sin 2x}$$

(c) $$\frac{dy}{dx} = 2\sqrt{y}\cos x \quad \text{(from part (a))}$$

$$= 2(2+\sin x)\cos x$$

$$= 4\cos x + 2\sin x\cos x$$

$$= 4\cos x + \sin 2x$$ ← same as part (b)

Monday, April 27, 2009
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Exercise Set 7.2

(4) $y = \ln(2+\sqrt{x})$

$$\frac{dy}{dx} = \frac{1}{2+\sqrt{x}}\frac{d}{dx}\left(2+\sqrt{x}\right) = \frac{1}{2+\sqrt{x}} \times \frac{1}{2\sqrt{x}}$$

$$= \boxed{\frac{1}{4\sqrt{x}+2x}}$$

(6) $y = \ln|x^3 - 7x^2 - 3|$

$$\frac{dy}{dx} = \frac{1}{x^3-7x^2-3}\frac{d}{dx}\left(x^3-7x^2-3\right)$$

$$= \boxed{\frac{3x^2-14x}{x^3-7x^2-3}}$$

(19) $y = \ln(\ln x)$

$$\frac{dy}{dx} = \frac{1}{\ln x}\frac{d}{dx}\ln x = \boxed{\frac{1}{x\ln x}}$$

(20) $y = \ln(\ln(\ln(x)))$

$$\frac{dy}{dx} = \frac{1}{\ln(\ln(x))}\frac{d}{dx}\ln(\ln(x))$$

use (19)

$$= \boxed{\frac{1}{\ln(\ln(x))} \times \frac{1}{x\ln x}}$$

(32) Note that, in this problem, you are asked to use logarithmic differentiation

logarithmic differentiation

$$y = \left(\frac{x-1}{x+1}\right)^{1/5} \qquad \ln y = \frac{1}{5}\left(\ln(x-1) - \ln(x+1)\right)$$

$$\Big\downarrow \frac{d}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{5}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

$$= \frac{1}{5}\,\frac{2}{(x-1)(x+1)}$$

$$\frac{dy}{dx} = y \times \frac{2}{5(x-1)(x+1)}$$

$$= \boxed{\sqrt[5]{\frac{x-1}{x+1}}\left(\frac{2}{5} \times \frac{1}{(x-1)(x+1)}\right)}$$

Exercise Set 7.3

27) $y = (x^3 - 2x)^{\ln x}$

$$\ln y = (\ln x)\left(\ln(x^3 - 2x)\right)$$

$$\Big\downarrow \frac{d}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\ln(x^3-2x) + (\ln x)\frac{1}{x^3-2x}(3x^2-2)$$

$$\boxed{\frac{dy}{dx} = (x^3-2x)^{\ln x}\left(\frac{1}{x}\ln(x^3-2x) + \frac{3x^2-2}{x^3-2x}\ln x\right)}$$

Exercise Set 7.5

6) $$\lim_{x\to 0}\frac{\sin 2x}{\sin 5x} \underset{\text{LHR}}{=} \lim_{x\to 0}\frac{2\cos 2x}{5\cos 5x} = \boxed{\frac{2}{5}}$$

10 $\displaystyle\lim_{x\to 0^+} \frac{\sin x}{x^2} \underset{\text{LHR}}{=} \lim_{x\to 0^+} \frac{\cos x}{2x} = \boxed{+\infty}$

Plugging in $x=0$ gives $\frac{1}{0}$. So, limit is either $+\infty$ or $-\infty$.
Because $x \to 0^+$, $2x > 0$ and $\frac{\cos x}{2x} > 0$

26 $\displaystyle\lim_{x\to +\infty} \left(1+\frac{a}{x}\right)^{bx}$

Let $y = \left(1+\frac{a}{x}\right)^{bx}$

$$\ln y = bx \ln\left(1+\frac{a}{x}\right)$$

$$= b\,\frac{\ln(1+a/x)}{1/x} \qquad \frac{0}{0}$$

$$\lim_{x\to +\infty} \ln y \underset{\text{LHR}}{=} \frac{b\,\frac{1}{1+\frac{a}{x}}\,a\left(\cancel{-\frac{1}{x^2}}\right)}{\cancel{-\frac{1}{x^2}}}$$

$$\ln \lim_{x\to\infty} y = \lim_{x\to\infty} \frac{ab}{1+\frac{a}{x}} = ab$$

$$\lim_{x\to\infty} y = \boxed{e^{ab}}$$

38 (a) L'Hôpital's Rule does not apply to the problem because it is of the

$$\frac{e^{\smile}}{0} = \frac{1}{0}$$

form which is <u>not</u> indeterminate

(b) From $\frac{1}{0}$, we know that the limit

is $+\infty$ or $-\infty$.

As $x \to 2^+$, $x^2-4 > 0$ and $\lim\limits_{x \to 2^+} = +\infty$

As $x \to 2^-$, $x^2-4 < 0$ and $\lim\limits_{x \to 2^-} = -\infty$

Because $\lim\limits_{x \to 2^+} \neq \lim\limits_{x \to 2^-}$, the limit $\boxed{\text{DNE}}$

48 (a) Let $y = x^{(\ln a)/(1+\ln x)}$

$$\ln y = \frac{\ln a}{1+\ln x} \ln x$$

$$\lim_{x \to \infty} \ln y = \ln a \frac{\cancel{\frac{1}{x}}}{\cancel{\frac{1}{x}}}$$

LHR

$$\lim_{x \to \infty} y = e^{\ln a} = \boxed{a}$$

50 $\sin x$ oscillates between -1 and 1 as $x \to \infty$
So, we can't directly conclude that

$$\lim_{x \to +\infty} \frac{2x - \sin x}{3x + \sin x}$$

is of any of the indeterminate form.

However, we can rewrite it as

$$\lim_{x \to +\infty} \frac{2 - \frac{\sin x}{x}}{3 + \frac{\sin x}{x}} = \frac{2-0}{3+0} = \boxed{\frac{2}{3}}$$

x

$$\lim_{x \to +\infty} \frac{\sin x}{x} = 0$$

we did this
in class using
the squeezing
theorem.