Sirindhorn International Institute of Technology
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Department of Common and Graduate Studies

MAS 116: Solution for Problem Set 3

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Course Title: MAS116 (Mathematics I)

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9:35 AM

Exercise Set 3.3

(2) $y = -3x^{12}$

Power rule

$$\frac{dy}{dx} = (-3)(12)x^{12-1} = \boxed{-36x^{11}}$$

(4) $y = \frac{1}{2}(x^4 + 7)$

$$\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(x^4+7) = \frac{1}{2}\left(\frac{d}{dx}x^4 + \frac{d}{dx}7\right)$$

Sum Rule

constant multiple rule

$$= \frac{1}{2}\left(\underbrace{4x^{4-1}}_{\text{Power rule}} + \underbrace{0}_{\text{derivative of a constant function is 0}}\right) = \frac{1}{2}\times 4x^3 = \boxed{2x^3}$$

(12) $f(x) = 7x^{-6} - 5\sqrt{x}$

$$f'(x) = 7\left(\frac{d}{dx}x^{-6}\right) - 5\left(\frac{d}{dx}\sqrt{x}\right) = 7(-6)\underbrace{x^{-6-1}}_{\text{Power rule}} - 5\underbrace{\frac{1}{2\sqrt{x}}}$$

linearity of differentiation

$$= \boxed{-42x^{-7} - \frac{5}{2\sqrt{x}}}$$

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

(18) $y = \frac{x^{3/2}+2}{x} = x^{\frac{1}{2}} + \frac{2}{x}$

$$y' = \frac{1}{2\sqrt{x}} + \underbrace{2(-1)x^{-1-1}}_{\text{power rule}} = \frac{1}{2\sqrt{x}} - 2x^{-2} = \frac{1}{2\sqrt{x}} - \frac{2}{x^2}$$

$$y'(1) = \frac{1}{2\sqrt{1}} - \frac{2}{1^2} = \frac{1}{2} - 2 = \boxed{-\frac{3}{2}}$$

(23) $y = (1-x)(1+x)(1+x^2)(1+x^4)$

$$= (1-x^2)(1+x^2)(1+x^4)$$

Repeated use of the formula

$$= (1-x^4)(1+x^4) \qquad (a+b)(a-b) = a^2-b^2$$

$$= 1-x^8$$

$$y' = \underset{\substack{\uparrow \\ \text{derivative of a constant function is 0}}}{0} - \underbrace{8x^{8-1}}_{\text{power rule}} = -8x^7$$

$$\left.\frac{dy}{dx}\right|_{x=1} = -8(1)^7 = \boxed{-8}$$

(40) (a) $y = 5x^2 - 4x + 7$

$$y' = 5(2)x^{2-1} - 4x^{1-1} + 0 = 10x - 4$$

$$y'' = 10x^{1-1} + 0 = 10$$

$$y''' = \boxed{0}$$

(b) $y = 3x^{-2} + 4x^{-1} + x$

$$y' = 3(-2)x^{-2-1} + 4(-1)x^{-1-1} + x^{1-1}$$

$$= -6x^{-3} - 4x^{-2} + 1$$

$$y'' = (-6)(-3)x^{-3-1} + (-4)(-2)x^{-2-1} + 0$$

$$= 18x^{-4} + 8x^{-3}$$

$$y''' = 18(-4)x^{-4-1} + 8(-3)x^{-3-1}$$

$$= \boxed{-72x^{-5} - 24x^{-4}}$$

(c) $y = ax^4 + bx^2 + c$ (a, b, c constant)

$$y' = a(4)x^{4-1} + b(2)x^{2-1} + 0$$

$$= 4a x^3 + 2bx$$

$$y'' = 4a(3)x^{3-1} + 2bx^{1-1} = 12ax^2 + 2b$$

$$y''' = 12a(2)x^{2-1} + 0 = \boxed{24ax}$$

66 (a) $f(x) = \frac{1}{x} = x^{-1}$

$$f'(x) = (-1)x^{-1-1} = (-1)x^{-2}$$

$$f''(x) = (-1)(-2)x^{-2-1} = (-1)^2(1\times 2)x^{-3}$$

Try to find some pattern

$$f'''(x) = (-1)^2(1\times 2)(-3)x^{-3-1} = (-1)^3(1\times 2\times 3)x^{-4}$$

$$f^{(n)}(x) = (-1)^n(1\times 2\times 3\times \cdots \times n)x^{-(n+1)}$$

$$= \boxed{\frac{(-1)^n(n!)}{x^{n+1}}}$$

(b) $f(x) = \frac{1}{x^2} = x^{-2}$

$$f'(x) = (-2)x^{-2-1} = (-2)x^{-3}$$

$$f''(x) = (-2)(-3)x^{-3-1} = (-2)(-3)x^{-4}$$

$$f'''(x) = (-2)(-3)(-4)x^{-4-1}$$

$$= (-2)(-3)(-4)x^{-5}$$

$$= (-1)^3(1\times 2\times 3\times 4)x^{-5}$$

Try to find some pattern

$$f^{(n)}(x) = (-1)^n(1\times 2\times 3\times \cdots \times n\times(n+1))\times x^{-(n+2)}$$

$$= \boxed{\frac{(-1)^n (n+1)!}{x^{n+2}}}$$

Exercise Set 3.4

(8) $f(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)(3x^3 + 27)$

$$f'(x) = \left(\frac{d}{dx}\left(\frac{1}{x} + \frac{1}{x^2}\right)\right)(3x^3+27)$$

$$+ \left(\frac{1}{x} + \frac{1}{x^2}\right)\frac{d}{dx}\left(3x^3 + 27\right)$$

$$= \left(-\frac{1}{x^2} - \frac{2}{x^3}\right)\left(3x^3 + 27\right) + \left(\frac{1}{x} + \frac{1}{x^2}\right)\left(9x^2\right)$$

$$= -3x - 6 - \frac{27}{x^2} - \frac{54}{x^3} + 9x + 9$$

$$= \boxed{6x + 3 - \frac{27}{x^2} - \frac{54}{x^3}}$$

(12) $y = \dfrac{3}{\sqrt{x} + 2}$

$$\frac{dy}{dx} = \frac{(\sqrt{x}+2)\cancel{\frac{d}{dx}(3)}^{0} - \left(\frac{d}{dx}(\sqrt{x}+2)\right)3}{(\sqrt{x}+2)^2}$$

$$= \frac{-3\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x}+2)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{-\frac{3}{2}}{(1+2)^2} = -\frac{3}{18} = \boxed{-\frac{1}{6}}$$

22b

$$F(x) = x\left(f(x) + g(x)\right)$$

$$F'(x) = \left(f(x) + g(x)\right) + x\left(f'(x) + g'(x)\right)$$

$$F'(\pi) = (10 + -3) + \pi(-1 + 2) = \boxed{7 + \pi}$$

35a

$$\frac{d}{dx}\left((2x+1)\left(1+\frac{1}{x}\right)\left(x^{-3}+7\right) \right)$$

$$= \boxed{2\left(1+\frac{1}{x}\right)\left(x^{-3}+7\right) + (2x+1)\left(-\frac{1}{x^2}\right)\left(x^{-3}+7\right) + (2x+1)\left(1+\frac{1}{x}\right)\left(-3x^{-4}\right)}$$