Sirindhorn International Institute of Technology
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MAS 116: Solution for Problem Set 2

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Course Title: MAS116 (Mathematics I)

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Exercise Set 2.3

6 $\lim_{x \to -\infty} f(x) = 7$ and $\lim_{x \to -\infty} g(x) = -6$

(a) $\lim_{x \to -\infty} \left(2 f(x) - g(x) \right)$

$= 2 \lim_{x \to -\infty} f(x) - \lim_{x \to -\infty} g(x)$

$= 2 \times 7 - (-6) = 14 + 6 = \boxed{20}$

(b) $6(7) + 7(-6) = 42 - 42 = \boxed{0}$

(c) $\boxed{+\infty}$

(d) $\boxed{-\infty}$

(e) $\sqrt[3]{7 \times (-6)} = \boxed{(-42)^{1/3}}$

(f) $\frac{(-6)}{(7)} = \boxed{-\frac{6}{7}}$

(g) $7 + (-6)\left(\lim_{x \to -\infty} \frac{1}{x} \right) = 7 + (-6)0 = \boxed{7}$

(h) $\left(\lim_{x \to -\infty} \frac{x}{2x+1} \right) \left(\frac{\lim_{x \to -\infty} f(x)}{\lim_{x \to -\infty} g(x)} \right) = \frac{1}{2} \times \frac{7}{(-6)}$

$= \boxed{-\frac{7}{12}}$

10 $\boxed{+\infty}$

16 $\deg(5x^2 + 7) = \deg(3x^2 - x) = 2$

Therefore, $\lim_{x \to +\infty} \frac{5x^2 + 7}{3x^2 - x} = \boxed{\frac{5}{3}}$

18 $\deg(3s^7 - 4s^5) = \deg(2s^7 + 1) = 7$

(18) $\deg(3s^7 - 4s^5) = \deg(2s^7 + 1) = 7$

Therefore, $$\lim_{x \to +\infty} \frac{3s^7 - 4s^5}{2s^7 + 1} = \frac{3}{2}$$

and $$\lim_{x \to +\infty} \sqrt[3]{\bigcirc} = \sqrt[3]{\lim_{x \to +\infty} \bigcirc}$$

$$= \boxed{\sqrt[3]{\frac{3}{2}}}$$

(24) $$\lim_{x \to +\infty} \frac{\sqrt{3x^4 + x}}{x^2 - 8} = \lim_{x \to +\infty} \frac{\sqrt{3x^4 + 8} \,/\, x^2}{(x^2 - 8) \,/\, x^2}$$

$$= \lim_{x \to +\infty} \frac{\sqrt{(3x^4 + 8)/x^4}}{1 - \frac{8}{x^2}} = \lim_{x \to +\infty} \frac{\sqrt{3 + \frac{8}{x^4}\to 0}}{1 - \frac{8}{x^2}\to 0}$$

$$= \boxed{\sqrt{3}}$$

(32) (a) $$\lim_{t \to -\infty} g(t) = \lim_{t \to -\infty} \frac{2 + 3t}{5t^2 + 6} = \boxed{0}$$

degree$(2 + 3t)$ $<$ degree$(5t^2 + 6)$

(b) $$\lim_{t \to +\infty} g(t) = \lim_{t \to +\infty} \frac{\sqrt{36t^2 - 100}}{5 - t}$$

$$= \lim_{t \to +\infty} \frac{\sqrt{36t^2 - 100} \,/\, t}{(5 - t) \,/\, t}$$

$$= \lim_{t \to +\infty} \frac{\sqrt{\frac{36t^2 - 100}{t^2}}}{\frac{5}{t} - 1}$$

$$= \lim_{t \to +\infty} \frac{\frac{5}{t} - 1 \cdot \sqrt{36 - \cancel{\frac{100}{t^2}}^{\,\to 0}}}{\cancel{\frac{5}{t}}^{\,\to 0} - 1} = \frac{\sqrt{36}}{-1} = \boxed{-6}$$

34 $\sqrt{x^2 - 3x} - x \times \frac{\sqrt{x^2 - 3x} + x}{\sqrt{x^2 - 3x} + x}$

$$= \frac{\cancel{x^2} - 3x - \cancel{x^2}}{\sqrt{x^2 - 3x} + x} = \frac{-3x}{\sqrt{x^2 - 3x} + x} \times \frac{1/x}{1/x}$$

$$= \frac{-3}{\sqrt{\frac{x^2 - 3x}{x^2}} + 1} = \frac{-3}{\sqrt{1 - \frac{3}{x}} + 1}$$

$$\lim_{x \to +\infty} \sqrt{x^2 - 3x} - x = \lim_{x \to +\infty} \frac{-3}{\sqrt{1 - \frac{3}{x}} + 1} \quad \left(\frac{3}{x} \to 0\right)$$

$$= \frac{-3}{\sqrt{1} + 1} = \boxed{-\frac{3}{2}}$$

36 $\left(\sqrt{x^2 + ax} - \sqrt{x^2 + bx}\right) \times \frac{\sqrt{x^2 + ax} + \sqrt{x^2 + bx}}{\sqrt{x^2 + ax} + \sqrt{x^2 + bx}}$

$$= \frac{(\cancel{x^2} + ax) - (\cancel{x^2} + bx)}{\sqrt{x^2 + ax} + \sqrt{x^2 + bx}} = \frac{(a - b)x}{\sqrt{} + \sqrt{}} \times \frac{1/x}{1/x}$$

$$= \frac{a - b}{\sqrt{\frac{x^2 + ax}{x^2}} + \sqrt{\frac{x^2 + bx}{x^2}}} = \frac{a - b}{\sqrt{1 + \frac{a}{x}} + \sqrt{1 + \frac{b}{x}}}$$

$$\lim_{x \to +\infty} \left(\sqrt{x^2+ax} - \sqrt{x^2+bx} \right) = \frac{a-b}{\sqrt{1}+\sqrt{1}} = \boxed{\frac{a-b}{2}}$$

(40) (a) $p(x) = x$, $q(x) = x$

(b) $p(x) = x$, $q(x) = x^2$

(c) $p(x) = x^2$, $q(x) = x$

(d) $p(x) = x+3$, $q(x) = x$

Exercise Set 2.5

(6) Because f is a continuous function, it is continuous at $x = 3$.

Therefore $\lim_{x \to 3} f(x) = f(3) = -2$

$$\lim_{x \to 3} \frac{f(x)}{g(x)} = \frac{\lim_{x \to 3} f(x)}{\lim_{x \to 3} g(x)} = \boxed{\frac{-2}{5}}$$

(8) Try $\boxed{f(x) = \frac{1}{x}}$

or

$$\boxed{g(x) = \begin{cases} 0, & x = 0 \\ \sin\frac{1}{x}, & x \neq 0 \end{cases}}$$

(12) $\boxed{\text{None}}$ (f is continuous everywhere in its domain)

(14) $\boxed{x = 2, -2}$

(18) $\boxed{x = 0, -4}$

20) x such that $x^4 + x = 0$

$$(x^3+1)x = 0$$

$$\boxed{x = 0, -1}$$

24) $\lim_{x \to -3^+} f(x) = \frac{k}{(-3)^2} = \frac{k}{9}$

$$f(3) = 9-(-3)^2 = 9-9=0$$

Need $\lim_{x \to -3^+} = f(3)$

That is $\frac{k}{9} = 0$. So, $\boxed{k=0}$

25) $\lim_{x \to 2^+} f(x) = 2^2+5 = 9$

$$f(2) = m(2+1)+k = 3m+k$$

f is continuous at $x=2$

So, need $9 = 3m+k$ ----- $(*)$

$\lim_{x \to -1^+} f(x) = m(-1+1)+k = k$

$$f(-1) = 2(-1)^3 + (-1) + 7$$

$$= -2 -1 + 7 = 4$$

So, need $k = 4$

f is continuous at $x=-1$

Plug in the value of k into $(*)$

We then have $9 = 3m+4$

$$m = \frac{5}{3}$$

Hence, $\boxed{k=4 \quad \text{and} \quad m = \frac{5}{3}}$

30) (a) $f(x)$ is undefined at $x = 2$

So, f is $\boxed{\text{not continuous at } x=2}$

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2+2x+4)}$$

$$= \frac{4}{4+4+4} = \frac{1}{3}$$

We can remove the discontinuity by defining

$$f(2) = \frac{1}{3}$$

So, discontinuity at $x=2$ is $\boxed{\text{removable}}$

(b) For $x < 2$, $f(x) = 2x-3 \leftarrow$ polynomial

For $x > 2$, $f(x) = x^2 \leftarrow$ polynomial

Therefore, $f(x)$ is continuous for $x \neq 2$.

$$\left.\begin{aligned} \lim_{x\to 2^-} f(x) &= 2(2)-3 = 1 \\ \lim_{x\to 2^+} f(x) &= 2^2 = 4 \end{aligned}\right\} \text{not equal}$$

So, $\lim_{x\to 2} f(x)$ does not exist.

This discontinuity can not be removed by redefining $f(2)$ because the limit does not exist at $x=2$.

Hence, f has a $\boxed{\text{nonremovable discontinuity at } x=2}$

Exercise Set 3.1

(14) (a) slope $= f'(x_0) = \lim_{h\to 0} \dfrac{f(x_0+h) - f(x_0)}{h}$

$$f(x_0+h)-f(x_0) = (x_0+h)^2+3(\cancel{x_0}+h)+\cancel{8}-(x_0^2+\cancel{3x_0}+\cancel{8})$$

$$= \cancel{x_0^2}+2x_0h+3h-\cancel{x_0^2}$$

$$= h(2x_0+3)$$

$$f'(x_0) = \lim_{h\to 0} 2x_0+3 = \boxed{2x_0+3}$$

(b) $f'(2) = 2\times 2+3 = \boxed{7}$

Exercise set 3.2

(10) $$f'(x) = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$$

$$= \frac{x^2-(x+h)^2}{h(x+h)^2x^2}$$

$$= \frac{-2x\cancel{h}-h^{\cancel{2}}}{\cancel{h}(x+h)^2x^2} = \frac{-(2x+h)}{(x+h)^2x^2}$$

$$= \frac{-2\cancel{x}}{x^2x^{\cancel{2}}} = \boxed{-\frac{2}{x^3}}$$

$f'(-1) = 2$

Tangent line at $x=a$

$$y = f'(a)(x-a)+f(a)$$

$a=-1$ gives

$$y = 2(x-(-1))+\frac{1}{(-1)^2}$$

$$= 2(x+1)+1$$

$$= \boxed{2x+3}$$

$$= \boxed{2x + 3}$$

23. (a) D
(b) F
(c) B
(d) C
(e) A
(f) E

Exercise Set 3.3

10. $\frac{1}{2\sqrt{x}} \quad -\frac{1}{x^2}$

64. $$f\left(\tfrac{1}{2}\right) = \frac{3}{4} \times \left(\frac{1}{2}\right)^2 = \frac{3}{16}$$

$$f\left(\tfrac{1}{2}+h\right) = \begin{cases} \left(\frac{1}{2}+h\right)^3 + \frac{1}{16}, & h < 0 \\ \frac{3}{4}\left(\frac{1}{2}+h\right)^2, & h > 0 \end{cases}$$

$$\lim_{h \to 0^+} \frac{f\left(\frac{1}{2}+h\right) - f\left(\frac{1}{2}\right)}{h} = \lim_{h \to 0^+} \frac{\frac{3}{4}\left(\frac{1}{2}+h\right)^2 - \frac{3}{16}}{h}$$

$$= \lim_{h \to 0^+} \frac{\frac{3}{4}\left(\cancel{\frac{1}{4}} + h + h^2\right) - \cancel{\frac{3}{16}}}{h}$$

$$\lim \quad \frac{3}{} \left(1 + h \right) = \frac{3}{}$$

$$= \lim_{h \to 0^+} \quad \frac{\ }{4}(\ \ ) \quad \frac{\ }{4}$$

$$\lim_{h \to 0^-} \frac{f(\frac{1}{2}+h) - f(\frac{1}{2})}{h} = \lim_{h \to 0^-} \frac{(\frac{1}{2}+h)^3 + \frac{1}{16} - \frac{3}{16}}{h}$$

$$(\tfrac{1}{2}+h)^3 = (\tfrac{1}{2}+h)^2(\tfrac{1}{2}+h)$$
$$= (\tfrac{1}{4}+h+h^2)(\tfrac{1}{2}+h)$$
$$= \tfrac{1}{8} + \tfrac{h}{2} + \tfrac{h^2}{2} + \tfrac{h}{4} + h^2 + h^3$$
$$= \tfrac{1}{8} + \tfrac{3}{4}h + \tfrac{3}{2}h^2 + h^3$$

$$= \lim_{h \to 0^-} \frac{3}{4} + \frac{3}{2}h + h^2$$

$$= \frac{3}{4}$$

They are equal!

Therefore

$$f'(\tfrac{1}{2}) = \lim_{h \to 0} \frac{f(\frac{1}{2}+h) - f(\frac{1}{2})}{h}$$
$$= \boxed{\frac{3}{4}}$$

Exercise 3.4

(6) $f'(x) = (-1-9x^2)(7+x^5) + (2-x-3x^3)(5x^4)$

$$= -24x^7 - 6x^5 + 10x^4 - 63x^2 - 7$$

(10) $f(x) = x^4 - x^2$

$f'(x) = 4x^3 - 2x$

(14) $$\frac{dy}{dx} = \frac{(x^2-5)(4) - (2x)(4x+1)}{(x^2-5)^2}$$

$$\frac{dy}{dx}\Big| = \frac{(-4)(4) - 2(5)}{\quad} = \frac{-16-10}{\quad}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{(-4)(4) - 2(5)}{(-4)^2} = \frac{-16-10}{16}$$

$$= \frac{-26}{16} = \boxed{-\frac{13}{8}}$$

Exercise 3.6

12 $$\frac{1}{2\sqrt{x^3-2x+5}} \times (3x^2-2)$$