Sirindhorn International Institute of Technology
Thammasat University
Department of Common and Graduate Studies

MAS 116: Solution for Problem Set 1

Semester/Year: 3/2008

Course Title: MAS116 (Mathematics I)

Instructor: Dr. Prapun Suksompong (prapun@siit.tu.ac.th)

Section 1.1: 4, 10, 32

4. **(a)** The natural domain of f is $x \neq -1$, and for g it is the set of all x. Whenever $x \neq -1$, $f(x) = g(x)$, but they have different domains.

(b) The domain of f is the set of all $x \geq 0$; the domain of g is the same.

10. **(a)** $g(3) = \dfrac{3+1}{3-1} = 2$; $g(-1) = \dfrac{-1+1}{-1-1} = 0$; $g(\pi) = \dfrac{\pi+1}{\pi-1}$; $g(-1.1) = \dfrac{-1.1+1}{-1.1-1} = \dfrac{-0.1}{-2.1} = \dfrac{1}{21}$;
$g(t^2-1) = \dfrac{t^2-1+1}{t^2-1-1} = \dfrac{t^2}{t^2-2}$

(b) $g(3) = \sqrt{3+1} = 2; g(-1) = 3; g(\pi) = \sqrt{\pi+1}; g(-1.1) = 3$; $g(t^2-1) = 3$ if $t^2 < 2$ and $g(t^2-1) = \sqrt{t^2-1+1} = |t|$ if $t^2 \geq 2$.

32. **(i)** $x = 0$ causes division by zero **(ii)** $g(x) = \sqrt{x}+1$ for $x \geq 0$

Section 1.3: 30, 34, 38, 40, 50, 54, 64, 65, 75

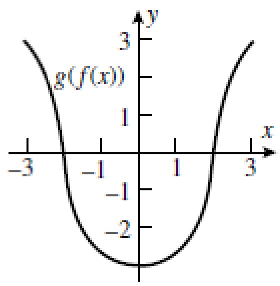
30. $(f+g)(x) = (2x^2+1)/[x(x^2+1)]$, all $x \neq 0$; $(f-g)(x) = -1/[x(x^2+1)]$, all $x \neq 0$; $(fg)(x) = 1/(x^2+1)$, all $x \neq 0$; $(f/g)(x) = x^2/(x^2+1)$, all $x \neq 0$

34. **(a)** $\sqrt{5s+2}$ **(b)** $\sqrt{\sqrt{x}+2}$ **(c)** $3\sqrt{5x}$ **(d)** $1/\sqrt{x}$
(e) $\sqrt[4]{x}$ **(f)** 0 **(g)** $1/\sqrt[4]{x}$ **(h)** $|x-1|$

38. $(f \circ g)(x) = \dfrac{x}{x^2+1}$, $x \neq 0$; $(g \circ f)(x) = \dfrac{1}{x} + x$, $x \neq 0$

40. $\dfrac{x}{x+1}$

50. Note that $g(f(-x)) = g(f(x))$, so $g(f(x))$ is even.



54. $\dfrac{w^2 + 6w - (x^2 + 6x)}{w - x} = w + x + 6$

$$\frac{(x+h)^2 + 6(x+h) - (x^2 + 6x)}{h} = \frac{2xh + h^2 + 6h}{h} = 2x + h + 6;$$

64. **(a)** $g(-x) = \dfrac{f(-x) + f(x)}{2} = \dfrac{f(x) + f(-x)}{2} = g(x)$, so g is even

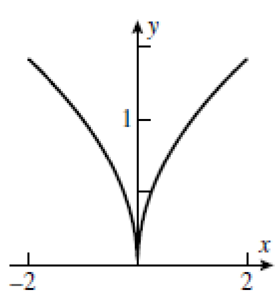
(b) $h(-x) = \dfrac{f(-x) - f(x)}{2} = -\dfrac{f(x) - f(-x)}{2} = -h(x)$, so h is odd

65. In Exercise 64 it was shown that g is an even funcction, and h is odd. Moreover by inspection $f(x) = g(x) + h(x)$ for all x, so f is the sum of an even function and an odd function.

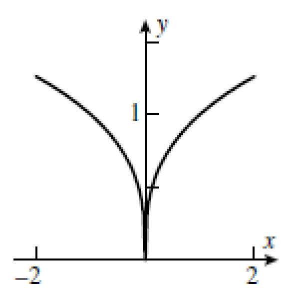
75. Yes, e.g. $f(x) = x^k$ and $g(x) = x^n$ where k and n are integers.

Section 1.4: 22

22. **(a)**



(b)

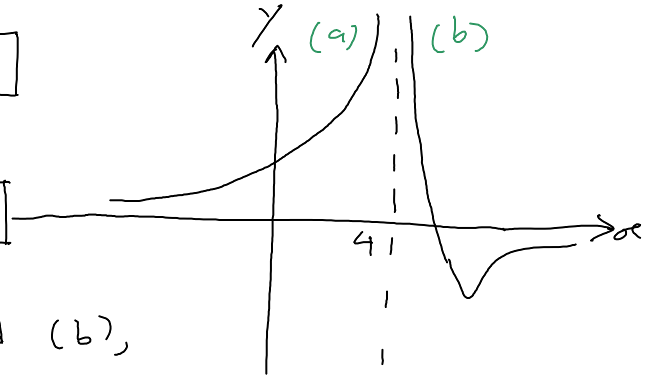


Exercise Set 2.1

Thursday, April 16, 2009
3:21 PM

(2) You can "read off" the answer from the accompanying figure.

(a) $\lim_{x \to 4^-} \phi(x) = \boxed{+\infty}$



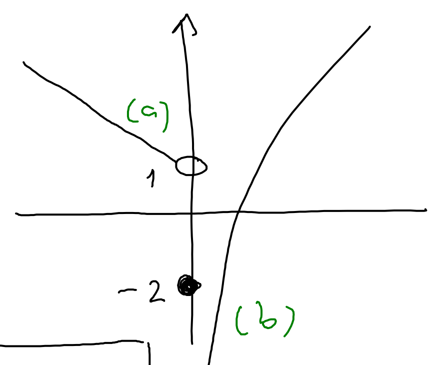
(b) $\lim_{x \to 4^+} \phi(x) = \boxed{+\infty}$

(c) From part (a) and (b),

$$\lim_{x \to 4} \phi(x) = \boxed{+\infty}$$

(d) $\phi(4)$ is $\boxed{\text{undefined}}$

(4) (a) $\lim_{x \to 0^-} f(x) = \boxed{1}$



(b) $\lim_{x \to 0^+} f(x) = \boxed{-\infty}$

(c) $\lim_{x \to 0} f(x)$ $\boxed{\text{does not exist}}$

because $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

(d) $f(0) = \boxed{-2}$ ← this is the black dot on the figure.

Wednesday, April 08, 2009
10:06 AM

Exercise Set 2.2

(8) After plugging in $t=-2$ into $\frac{t^3+8}{t+2}$

we get $\frac{0}{0}$. This suggests that both the nominator and denominator have a factor of $(t+2)$.

$$\frac{t^3+8}{t+2} = \frac{(t+2)(t^2-2t+4)}{(t+2)\times 1}$$

$$= t^2-2t+4, \quad t \neq -2$$

for $\lim_{t\to -2} f(t)$, we don't care about the value of $f(t)$ at $t=-2$.

$$\lim_{t\to -2} \frac{t^3+8}{t+2} = \lim_{t\to -2} t^2-2t+4$$

$$= (-2)^2 - 2(-2) + 4$$

$$= 4 + 4 + 4 = \boxed{12}$$

(10) Again, after plugging in $x=2$ into

$$\frac{x^2-4x+4}{x^2+x-6},$$

we get $\frac{0}{0}$. So, try factoring.

$$\frac{x^2-4x+4}{x^2+x-6} = \frac{(x-2)(x-2)}{(x+3)(x-2)}$$

$$= \frac{x-2}{x+3}, \quad x \neq 2$$

So, $\lim_{x\to 2} \frac{x^2-4x+4}{x^2+x-6} = \lim_{x\to 2} \frac{x-2}{x+3} = \frac{2-2}{2+3}$

$$= \frac{0}{5} = \boxed{0}$$

(16) Plugging $x = 3$ into $\frac{x}{x-3}$ gives $\frac{3}{0}$.

This suggests that $\boxed{\text{the limit does not exist}}$. In particular, we have the case of infinite limit. We now want to determine whether the limit is $+\infty$ or $-\infty$.

We want to find $\lim\limits_{x \to 3^-} f(x)$; so

we consider x near 3 but $x < 3$

which implies

$$\frac{3}{x} > 1$$

$$-\frac{3}{x} < -1$$

$$1 - \frac{3}{x} < 0$$

As $x \to 3^-$, we then have

$$1 - \frac{3}{x} \to 0^-$$

Therefore, $$\lim_{x \to 3^-} \frac{x}{x-3} = \lim_{x \to 3^-} \frac{1}{1 - \frac{3}{x}}$$

$$= \boxed{-\infty}$$

(20) Plugging in $x = 2$ into $\frac{x}{x^2-4}$ gives $\frac{2}{0}$.

This suggests that $\boxed{\text{the limit does not exist}}$. In particular, we have the case of infinite limit. We now want to determine whether the limit is $+\infty$ or $-\infty$.

For x near 2 $\quad x > 0$

and

$x^2 - 4 > 0$ if $x > 2$

$x^2 - 4 < 0$ if $x < 2$

So, $\dfrac{x}{x^2-4} > 0$ if $x > 2$

and $\dfrac{x}{x^2-4} < 0$ if $x < 2$

Hence, $\displaystyle\lim_{x \to 2^+} \frac{x}{x^2-4} = +\infty$

$\displaystyle\lim_{x \to 2^-} \frac{x}{x^2-4} = -\infty$

30 $\quad \dfrac{4-y}{2-\sqrt{y}} = \dfrac{(2-\sqrt{y})(2+\sqrt{y})}{2-\sqrt{y}} = 2+\sqrt{y}, \quad y \neq 4$

$\displaystyle\lim_{x \to 4} \frac{4-y}{2-\sqrt{y}} = \lim_{x \to 4} 2+\sqrt{y} = 2+\sqrt{4} = \boxed{4}$

36 $\quad \dfrac{1}{x} + \dfrac{1}{x^2} = \dfrac{x+1}{x^2}$

Plugging in $x = 0$ gives $\frac{1}{0}$.

So, limit $\boxed{\text{does not exist}}$

To find out whether the limit is $+\infty$ or $-\infty$, we note that for $\displaystyle\lim_{x \to 0^-} f(x)$, we consider the x near 0 that is < 0.

So $\left.\begin{array}{c} x+1>0 \\ \text{and} \\ x^2>0 \end{array}\right\} \Rightarrow \dfrac{x+1}{x^2}>0$

$$\lim_{x\to 0^+} \frac{x+1}{x^2} = \boxed{+\infty}$$

(38) $$\frac{\sqrt{x^2+4}-2}{x} = \frac{\sqrt{x^2+4}-2}{x} \cdot \frac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2}$$

$$= \frac{x^2+\cancel{4}-\cancel{4}}{x\left(\sqrt{x^2+4}+2\right)} = \frac{x^{\cancel{2}}}{\cancel{x}\left(\sqrt{x^2+4}+2\right)}$$

$$\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x} = \frac{0}{\sqrt{0+4}+2} = \frac{0}{4} = \boxed{0}$$

(40) $$f(x) = \begin{cases} \dfrac{(a+b)x+(a-b)x}{2x} & x>0 \\[2ex] \dfrac{(a+b)x+(a-b)(-x)}{2x} & x<0 \end{cases}$$

$$= \begin{cases} \dfrac{\cancel{2}a\cancel{x}}{\cancel{2x}}, & x>0 \\[2ex] \dfrac{\cancel{2}b\cancel{x}}{\cancel{2x}}, & x<0 \end{cases} = \begin{cases} a & x>0 \\ b & x<0 \end{cases}$$

(a) $\displaystyle\lim_{x\to 0^-} f(x) = \boxed{b}$

(b) $\displaystyle\lim_{x\to 0^+} f(x) = \boxed{a}$

(c) Need $\displaystyle\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

So, can use $\boxed{\text{any pair } (a,b) \text{ such that } a=b.}$