Sirindhorn International Institute of Technology
Thammasat University
Department of Common and Graduate Studies

MAS 116: Midterm Examination

Solution for Set B

COURSE : MAS116 (Mathematics I)
DATE : April 22, 2009
SEMESTER : 3/2008
INSTRUCTOR : Dr. Prapun Suksompong
TIME : 13:30-16:30
PLACE : RS 414

Name		ID	
Section	1	Seat	

Instructions:

1. Including this cover page, this exam has 9 pages.
2. **Read the questions carefully**.
3. Write your **name and ID** on each page of your examination paper.
4. Write all your work in the space provided. You may not get full credit even when your answer is right without all of your work written down.
5. Closed book. Closed notes. No calculator.
6. Allocate your time wisely.
7. Do not cheat. The use of communication devices including mobile phones is prohibited in the examination room.
8. Do not panic.

Good Luck!

Name: ID:

1. (25 pt) Let $f(x)=\sqrt{x+2}$ and $g(x)=\sqrt{x+2}$.

   a. (3 pt) Find the domain of f.

   " 2 " is inside $\sqrt{\ }$.
   Therefore, we must have $x+2 \geq 0$
   $x \geq -2$

   So, domain is $\boxed{D_f = \{x : x \geq -2\}}$

   b. (2 pt) Find the domain of g.

   Same as the domain of f. $\boxed{D_g = \{x : x \geq -2\}}$

   c. (20 pt) Find the formula and domain of the following functions

$$f+g,\quad f-g,\quad fg,\quad f/g,\quad f\circ g,\quad g\circ f$$

Put your answers in the table below

	Formula	Domain
$f+g$	$2\sqrt{x+2}$	$\{x : x \geq -2\}$
$f-g$	0	$\{x : x \geq -2\}$
fg	$x+2$	$\{x : x \geq -2\}$
f/g	1	$\{x : x > -2\}$
$f\circ g$	$\sqrt{\sqrt{x+2}+2}$	$\{x : x \geq -2\}$
$g\circ f$	$\sqrt{\sqrt{x+2}+2}$	$\{x : x \geq -2\}$

$D_f \cap D_g$

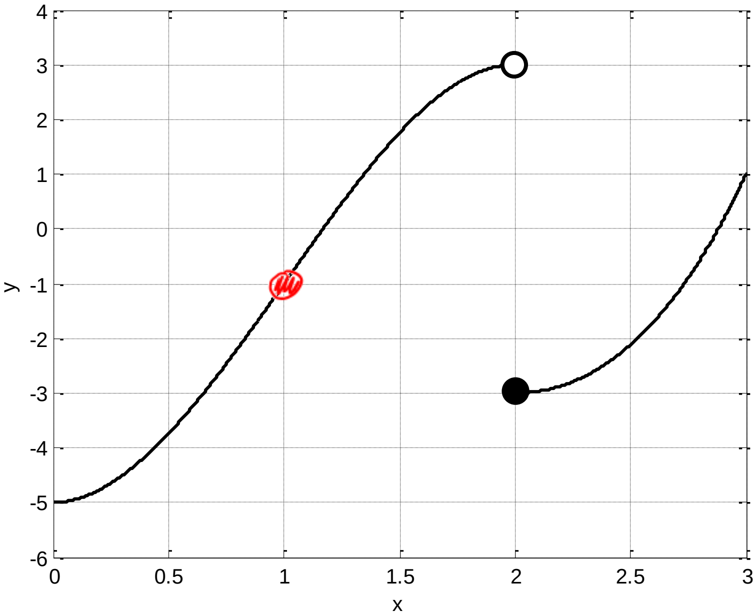
need to exclude the point that makes $g(x)=0$. In this case, only $x=-2$ needs to be excluded.

For any x in the domain of g, $f(x)$ is ≥ 0. Therefore, $f(x)$ is in the domain of f. Hence, all x in D_g is in the domain of $f\circ g$

By symmetry, the domain of $g\circ f$ is the same as the domain of $f\circ g$.

Name: ID:

2. (13 pt) Consider the function *f* plotted below.



Answer the following questions:

a. (1 pt) $f(1)$ = -1

b. (1 pt) $\lim_{x\to 1^-} f(x)$ = -1

c. (1 pt) $\lim_{x\to 1^+} f(x)$ = -1

Read these off from the plot above.

d. (1 pt) $\lim_{x\to 1} f(x)$ = -1 because $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = -1$

e. (3 pt) Is the function *f* continuous at $x=1$? Justify your answer.

Yes, because $f(1) = \lim_{x\to 1} f(x)$

f. (1 pt) $f(2)$ = -3

g. (1 pt) $\lim_{x\to 2^+} f(x)$ = -3

h. (1 pt) $\lim_{x\to 2^-} f(x)$ = 3

Read these off from the plot above.

i. (3 pt) Is the function *f* continuous at $x=2$? Justify your answer.

$\boxed{\text{No}}$ because $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$

which means $\lim_{x\to 2} f(x)$ does not exist.

Name: ID:

3. (20 pt) Consider the function f defined by the following formula:

$$f(x)=\begin{cases} x^3-3x^2+1, & x\geq 2 \\ -2x^3+6x^2-5, & x<2 \end{cases}$$

Answer the questions below.

a. (2 pt) $f(1)$ =

Because $1<2$, we use the formula $-2x^3+6x^2-5$.

$f(1) = -2(1)^3+6\times 1^2-5 = -2+6-5 = \boxed{-1}$

b. (2 pt) $\lim_{x\to 1^-} f(x)$ = $\boxed{-1}$

c. (2 pt) $\lim_{x\to 1^+} f(x)$ = $\boxed{-1}$

d. (2 pt) $\lim_{x\to 1} f(x)$ = $\boxed{-1}$

Because $-2x^3+6x^2-5$ is a polynomial, it is continuous and thus we can evaluate the limit by substitution which is the same as $f(1)$.

e. (3 pt) Is the function f continuous at $x=1$? Justify your answer.

$\boxed{\text{Yes}}$ by the explanation above or by the fact that the answer in part (a) is the same as the answer in part (d).

f. (2 pt) $f(2)$ =

$2^3-3\times 2^2+1 = 8-12+1 = \boxed{-3}$

g. (2 pt) $\lim_{x\to 2^+} f(x)$ = $\boxed{-3}$

the "+" sign means we consider $x>2$.

Therefore, we use the formula x^3-3x^2+1 which is the same as part (f) above.

h. (2 pt) $\lim_{x\to 2^-} f(x)$ =

The "-" sign means we consider $x<2$.

Therefore, we use the formula $-2x^3+6x^2-5$. Substitution of $x=2$ gives

$-2\times 8+6\times 4-5$

$=-16+24-5$

$=\boxed{3}$

i. (3 pt) Is the function f continuous at $x=2$? Justify your answer.

$\boxed{\text{No}}$, because $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$

which means $\lim_{x\to 2} f(x)$ does not exist.

4. (22 pt) Evaluate the following limits

a. (2 pt) $\lim_{x\to 2} x^2+x+8$ $= 2^2+2+8 = \boxed{14}$

This is a polynomial so it is continuous and the limit can be found by substitution.

b. (2 pt) $\lim_{x\to 1} \frac{x^2+x}{8x}$ $= \frac{1^2+1}{8} = \frac{2}{8} = \boxed{\frac{1}{4}}$

This is a rational function and the point $x=1$ is in it domain. Therefore, limit at $x=1$ can be found by substitution.

c. (2 pt) $\lim_{x\to 0} \frac{x^2+x}{8x}$

Plugging in $x=0$ gives $\frac{0}{0}$. So need some manipulation.

$\frac{x^2+x}{8x} = \frac{\cancel{x}(x+1)}{8\cancel{x}}$

$\lim_{x\to 0} \frac{x+1}{8} = \boxed{\frac{1}{8}}$

d. (2 pt) $\lim_{x\to 0^+} \frac{x^2+8}{8x}$

Plugging in $x=0$ gives $\frac{8}{0}$. So, limit is ∞ or $-\infty$.

check the sign: Because we are finding $\lim_{x\to 0^+}$, we consider $x>0$. Therefore, $\frac{x^2+8}{8x} > 0$ which implies $\lim_{x\to 0^+} \frac{x^2+8}{8x} = \boxed{+\infty}$

e. (1 pt) $\lim_{x\to 0} \frac{x^2+8}{8x}$

First, we find $\lim_{x\to 0^-}$ which means we consider $x<0$. In which case $\frac{x^2+8}{8x} < 0$. So, $\lim_{x\to 0^-} \frac{x^2+8}{8x} = -\infty$.

Because $\lim_{x\to 0^+} \neq \lim_{x\to 0^-}$, the limit $\boxed{\text{does not exist}}$

f. (1 pt) $\lim_{x\to 0^+} \frac{x^2+8}{8x(x-1)}$

$= \boxed{-\infty}$

In part (e), we get $+\infty$. Here, the $(x-1)$ changes the sign of our answer because around $x=0$, we have $x-1<0$.

g. (1 pt) $\lim_{x\to 0^+} \frac{x^2+8}{8x(x-1)(x+1)}$ $= \boxed{-\infty}$

In part (f), we get $-\infty$. Here, the $(x+1)$ term does not affect the sign because around $x=0$, we have $x+1>0$.

h. (1 pt) $\lim_{x\to 0^+} \frac{\cancel{x^2-1}}{8x\cancel{(x-1)(x+1)}}$ $= \boxed{+\infty}$

$= \lim_{x\to 0^+} \frac{1}{8x}$

i. (2 pt) $\lim_{x\to 8} \frac{x-8}{x^2-7x-8}$ $= \lim_{x\to 8} \frac{\cancel{x-8}}{\cancel{(x-8)}(x+1)} = \lim_{x\to 8} \frac{1}{x+1} = \boxed{\frac{1}{9}}$

j. (2 pt) $\lim_{x\to 8} \sqrt{\frac{x^2-7x-8}{x-8}}$ $= \sqrt{\lim_{x\to 8} \bigcirc} = \sqrt{\frac{1}{\lim_{x\to 8} \bigcirc}} = \sqrt{\frac{1}{\frac{1}{9}}} = \sqrt{9}$

$= 3$

k. (2 pt) $\lim_{x\to 8^+} \frac{8|x-8|}{x-8}$ $= \lim_{x\to 8^+} \frac{8\cancel{(x-8)}}{\cancel{x-8}}$

This means we consider $x > 8$.

$= \boxed{8}$

Recall that

$$|x-a| = \begin{cases} x-a & x \geq a \\ -(x-a) & x < a \end{cases}$$

l. (2 pt) $\lim_{x\to 8} \frac{8|x-8|}{x-8}$ $\boxed{\text{does not exist}}$ because

$\lim_{x\to 8^-} \frac{8|x-8|}{x-8} = \lim_{x\to 8^-} \frac{8(-\cancel{(x-8)})}{\cancel{x-8}} = -8 \neq \lim_{x\to 8^+} \frac{8|x-8|}{x-8}$

m. (2 pt) $\lim_{x\to 8} e^{\frac{x-8}{x^2-7x-8}}$ $= e^{\lim_{x\to 8} \bigcirc} = e^{\frac{1}{9}}$

e^x is a continuous function.

5. Evaluate the following limits

a. (2 pt) $\lim_{x\to\infty}\frac{8x+8}{x+8} = \frac{8}{1} = \boxed{8}$

Deg(nominator) = Deg(denominator)
So, use the leading coefficients.

b. (2 pt) $\lim_{x\to\infty}\frac{8x+8}{x^2+8} = 0$

Deg(nominator) < Deg(denominator)

c. (2 pt) $\lim_{x\to\infty}\frac{8x+8}{|x+8|+x} = \lim_{x\to\infty}\frac{8x+8}{x+8+x} = \lim_{x\to\infty}\frac{8x+8}{2x+8} = \frac{8}{2} = \boxed{4}$

$x\to\infty$
So, $x > -8$

d. (1 pt) $\lim_{x\to\infty}\frac{8}{\sqrt{x^2+8}+x} = \lim_{x\to\infty}\frac{\frac{8}{x}\to 0}{\sqrt{1+\frac{8}{x^2}}+1} = \frac{0}{2} = \boxed{0}$

Remark: For rational function, we divide each term in the numerator and denominator by the highest power of x that occurs in the denominator.
In this case, the function is not rational, but we observe that $\sqrt{x^2} = x$ and hence we try deviding the numerator and denumerator by x.

e. (1 pt) $\lim_{x\to\infty}\frac{8x+8}{\sqrt{x^2+8x+8}+x}$

$$= \lim_{x\to\infty}\frac{8+\frac{8}{x}}{\sqrt{1+\frac{8}{x}+\frac{8}{x^2}}+1}$$

$$= \frac{8}{1+1} = \frac{8}{2} = \boxed{4}$$

f. (1 pt) $\lim_{x\to\infty} \sqrt{x^2+8} - x$

$$\sqrt{x^2+8} - x = \left(\sqrt{x^2+8} - x\right) \times \frac{\sqrt{x^2+8}+x}{\sqrt{x^2+8}+x}$$

$$= \frac{x^2+8-x^2}{\sqrt{x^2+8}+x} = \frac{8}{\sqrt{x^2+8}+x}$$

$$\lim_{x\to\infty} \frac{8}{\sqrt{x^2+8}+x} = \boxed{0}$$

from part (d)

g. (1 pt) $\lim_{x\to\infty} \sqrt{x^2+8x+8} - x$

$$\sqrt{x^2+8x+8} - x = \left(\sqrt{x^2+8x+8} - x\right) \times \frac{\sqrt{x^2+8x+8}+x}{\sqrt{x^2+8x+8}+x}$$

$$= \frac{8x+8}{\sqrt{x^2+8x+8}+x}$$

$$\lim_{x\to\infty} \frac{8x+8}{\sqrt{\quad}+x} = \boxed{4}$$

from part (e).

Name: ID:

6. (10 pt) Consider the function

$$f(x) = x^2 + 8$$

a. (5 pt) Use the *definition of the derivative alone* to find the derivative of *f*. No point will be given for answers derived from differentiation rules.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{((x+h)^2 + \cancel{8}) - (x^2 + \cancel{8})}{h}$$

$$= \lim_{h \to 0} \frac{2x\cancel{h} + h^{\cancel{2}}}{\cancel{h}} = \boxed{2x}$$

b. (5 pt) Find the equation of the tangent line to the graph of *f* at *x* = 4.

Equation of the tangent line at x_0:

$$y - f(x_0) = f'(x_0)(x - x_0)$$

Here,

$x_0 = 4$.

$$f(x_0) = f(4) = 4^2 + 8 = 16 + 8 = 24$$

$$f'(x_0) = f'(4) = 2(4) = 8$$

from part (a)

So, tangent line:

$$y - 24 = 8(x - 4)$$

$$y - 24 = 8x - 32$$

$$\boxed{y = 8x - 8}$$