Sirindhorn International Institute of Technology
Thammasat University
Department of Common and Graduate Studies

MAS 116: Midterm Examination

COURSE : MAS116 (Mathematics I)
DATE : April 22, 2009
SEMESTER : 3/2008
INSTRUCTOR : Dr. Prapun Suksompong
TIME : 13:30-16:30
PLACE : RS 414

Answers for Set A

Name		ID	
Section	1	Seat	

Instructions:

1. Including this cover page, this exam has 9 pages.
2. **Read the questions carefully**.
3. Write your **name and ID** on each page of your examination paper.
4. Write all your work in the space provided. You may not get full credit even when your answer is right without all of your work written down.
5. Closed book. Closed notes. No calculator.
6. Allocate your time wisely.
7. Do not cheat. The use of communication devices including mobile phones is prohibited in the examination room.
8. Do not panic.

Good Luck!

1. (25 pt) Let $f(x)=\sqrt{x+1}$ and $g(x)=\sqrt{x+1}$.
   a. (3 pt) Find the domain of f.

Need $x+1 \geq 0$

$x \geq -1$

Domain $= \boxed{[-1, \infty)}$

   b. (2 pt) Find the domain of g.

The expression for $g(x)$ is the same as that of $f(x)$.

So, same domain : $\boxed{[-1, \infty)}$

   c. (20 pt) Find the formula and domain of the following functions

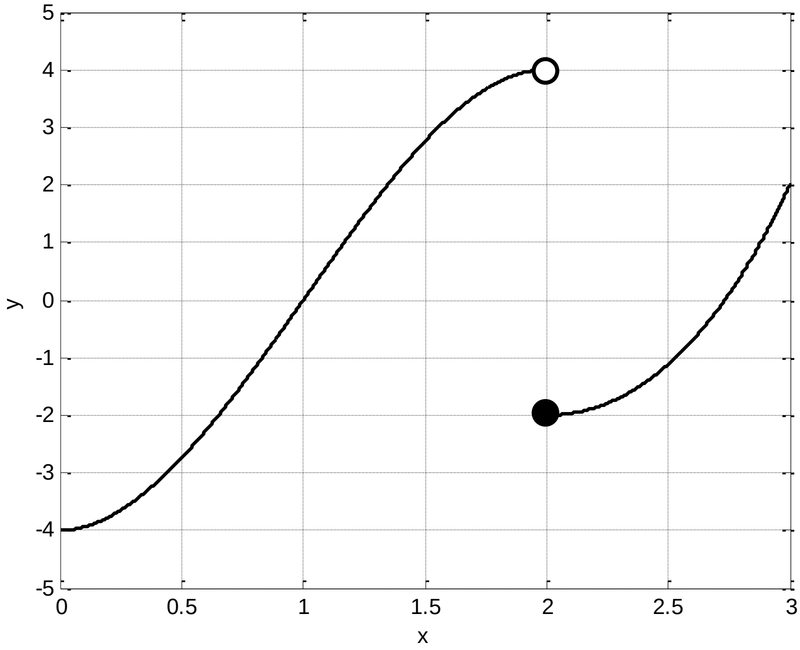
$$f+g, \quad f-g, \quad fg, \quad f/g, \quad f\circ g, \quad g\circ f$$

Put your answers in the table below

	Formula	Domain
$f+g$	1 $2\sqrt{x+1}$	2 $[-1, \infty)$
$f-g$	1 0	2 $[-1, \infty)$
fg	1 $x+1$	2 $[-1, \infty)$
f/g	1 1	3 $(-1, \infty)$
$f\circ g$	1 $\sqrt{\sqrt{x+1}+1}$	3 $[-1, \infty)$
$g\circ f$	1 $\sqrt{\sqrt{x+1}+1}$	2 $[-1, \infty)$

need to exclude the point $x=-1$ which make $g(x)=0$

2. (13 pt) Consider the function *f* plotted below.



Answer the following questions:

a. (1 pt) $f(1)$ = 0

b. (1 pt) $\lim\limits_{x\to 1^-} f(x)$ = 0

c. (1 pt) $\lim\limits_{x\to 1^+} f(x)$ = 0

d. (1 pt) $\lim\limits_{x\to 1} f(x)$= 0

e. (3 pt) Is the function *f* continuous at $x=1$? Justify your answer.

Yes, because (1) $f(1)$ is defined, (2) $\lim\limits_{x\to 1} f(x)$ exists and (3) $f(1) = \lim\limits_{x\to 1} f(x)$.

f. (1 pt) $f(2)$ = -2

g. (1 pt) $\lim\limits_{x\to 2^+} f(x)$ = -2

h. (1 pt) $\lim\limits_{x\to 2^-} f(x)$ = 4

$\Rightarrow \lim\limits_{x\to 2} f(x)$ DNE

i. (3 pt) Is the function *f* continuous at $x=2$? Justify your answer.

No, because $\lim\limits_{x\to 2} f(x)$ does not exist

3. (20 pt) Consider the function f defined by the following formula:

$$f(x)=\begin{cases} x^3-3x^2+2, & x\geq 2 \\ -2x^3+6x^2-4, & x<2 \end{cases}$$

Answer the questions below.

a. (2 pt) $f(1)$ = 0

b. (2 pt) $\lim_{x\to 1^-} f(x)$ = 0

c. (2 pt) $\lim_{x\to 1^+} f(x)$ = 0

d. (2 pt) $\lim_{x\to 1} f(x)$ = 0

e. (3 pt) Is the function f continuous at $x=1$? Justify your answer.

Yes, because $f(1) = \lim_{x\to 1} f(x)$

f. (2 pt) $f(2)$ = -2

g. (2 pt) $\lim_{x\to 2^+} f(x)$ = -2

$\Rightarrow \lim_{x\to 2} f(x)$ DNE

h. (2 pt) $\lim_{x\to 2^-} f(x)$ = 4

i. (3 pt) Is the function f continuous at $x=2$? Justify your answer.

No, because $\lim_{x\to 2} f(x)$ DNE

4. (22 pt) Evaluate the following limits

a. (2 pt) $\lim_{x\to 2} x^2+x+7$ $= 13$

b. (2 pt) $\lim_{x\to 1} \frac{x^2+x}{7x}$ $= \frac{2}{7}$

c. (2 pt) $\lim_{x\to 0} \frac{x^2+x}{7x}$ $= \frac{1}{7}$

d. (2 pt) $\lim_{x\to 0^+} \frac{x^2+7}{7x}$ $= +\infty$

e. (1 pt) $\lim_{x\to 0} \frac{x^2+7}{7x}$ $=$ DNE

$\lim_{x\to 0^-} \frac{x^2+7}{7x} = -\infty$

f. (1 pt) $\lim_{x\to 0^+} \frac{x^2+7}{7x(x-1)}$ $= -\infty$

g. (1 pt) $\lim_{x\to 0^+} \frac{x^2+7}{7x(x-1)(x+1)}$ $= -\infty$

h. (1 pt) $\lim\limits_{x \to 0^+} \dfrac{x^2-1}{7x(x-1)(x+1)}$ $= +\infty$

i. (2 pt) $\lim\limits_{x \to 7} \dfrac{x-7}{x^2-6x-7}$ $= \dfrac{1}{8}$

j. (2 pt) $\lim\limits_{x \to 7} \sqrt{\dfrac{x^2-6x-7}{x-7}}$ $= \sqrt{8}$

k. (2 pt) $\lim\limits_{x \to 7^+} \dfrac{7|x-7|}{x-7}$ $= 7$

l. (2 pt) $\lim\limits_{x \to 7} \dfrac{7|x-7|}{x-7}$ $=$ DNE

$\lim\limits_{x \to 7^-} \dfrac{7|x-7|}{x-7} = -7$

m. (2 pt) $\lim\limits_{x \to 7} e^{\frac{x-7}{x^2-6x-7}}$ $= e^{\frac{1}{8}}$

5. (10 pt) Evaluate the following limits

a. (2 pt) $\lim_{x\to\infty} \frac{7x+7}{x+7}$ $= 7$

b. (2 pt) $\lim_{x\to\infty} \frac{7x+7}{x^2+7}$ $= 0$

c. (2 pt) $\lim_{x\to\infty} \frac{7x+7}{|x+7|+x}$ $= \frac{7}{2}$

d. (1 pt) $\lim_{x\to\infty} \frac{7}{\sqrt{x^2+7}+x}$ $= 0$

e. (1 pt) $\lim_{x\to\infty} \frac{7x+7}{\sqrt{x^2+7x+7}+x}$ $= \frac{7}{2}$

f. (1 pt) $\lim_{x\to\infty} \sqrt{x^2+7} - x$ $= 0$

g. (1 pt) $\lim_{x\to\infty} \sqrt{x^2+7x+7} - x$ $= \frac{7}{2}$

6. (10 pt) Consider the function

$$f(x) = x^2 + 4$$

a. (5 pt) Use the *definition of the derivative alone* to find the derivative of *f*. No point will be given for answers derived from differentiation rules.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 4 - (x^2 + 4)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\cancel{x^2} + 2xh + h^2 - \cancel{x^2}\right)$$

$$= \lim_{h \to 0} 2x + h = \boxed{2x}$$

b. (5 pt) Find the equation of the tangent line to the graph of *f* at *x* = 4.

$$y - f(x_0) = f'(x_0)(x - x_0)$$

Here,

$x_0 = 4$

$f(x_0) = 4^2 + 4$

$= 20$

$f'(x_0) = 2 \times 4 = 8$

$$y - 20 = 8(x - 4)$$

$$\boxed{y = 8x - 12}$$