Name ________________________ ID3 _ _ _

**ECS 452: Digital Communication Systems** **2020/2**

# Additional Examples 1

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**Problem 1.** These codes cannot be Huffman codes. Why?

(a) $\{00, 01, 10, 110\}$

(b) $\{01, 10\}$

(c) $\{0, 01\}$

**Problem 2.** Construct a random variable $X$ (by specifying its pmf) whose corresponding Huffman code is $\{0, 10, 11\}$.

**Problem 3.** Consider a BSC whose crossover probability for each bit is $p = 0.35$. Suppose $P[X = 0] = 0.45$.

(a) Draw the channel diagram.

(b) Find the channel matrix $\mathbf{Q}$.

(c) Find the joint pmf matrix $\mathbf{P}$.

(d) Find the row vector $\underline{\mathbf{q}}$ which contains the pmf of the channel output $Y$.

(e) Analyze the performance of all four reasonable detectors for this binary channel. Complete the table below:

| $\hat{x}(y)$ | $\hat{x}(0)$ | $\hat{x}(1)$ | $P(\mathcal{C})$ | $P(\mathcal{E})$ |
|---|---|---|---|---|
| $y$ | | | | |
| $1-y$ | | | | |
| $1$ | | | | |
| $0$ | | | | |

**Problem 4.** Consider a BAC whose $Q(1|0) = 0.35$ and $Q(0|1) = 0.55$. Suppose $P[X = 0] = 0.4$.

(a) Draw the channel diagram.

(b) Find the joint pmf matrix $\mathbf{P}$.

(c) Find the row vector $\underline{\mathbf{q}}$ which contains the pmf of the channel output $Y$.

(d) Analyze the performance of all four reasonable detectors for this binary channel. Complete the table below:

| $\hat{x}(y)$ | $\hat{x}(0)$ | $\hat{x}(1)$ | $P(\mathcal{C})$ | $P(\mathcal{E})$ |
|---|---|---|---|---|
| $y$ | | | | |
| $1-y$ | | | | |
| $1$ | | | | |
| $0$ | | | | |

**Problem 5.** Consider a DMC whose $\mathcal{X} = \{1,2,3\}$, $\mathcal{Y} = \{1,2,3\}$, and $\mathbf{Q} = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$.

Suppose the input probability vector is $\underline{\mathbf{p}} = [0.2, 0.4, 0.4]$.

(a) Find the joint pmf matrix $\mathbf{P}$.

(b) Find the row vector $\underline{\mathbf{q}}$ which contains the pmf of the channel output $Y$.

(c) Find the following probabilities:

(i) $P[X = 1]$

(ii) $P[Y = 2]$

(iii) $P[X = 1, Y = 2]$

(iv) $P[Y = 2 | X = 1]$

(v) $P[X = 1 | Y = 2]$

(vi) Find the error probability of the naive decoder.

(vii) Find the error probability of the (DIY) decoder $\hat{x}(y) = 4 - y$.

**Problem 6.** ***Optimal code lengths that require one bit above entropy***: The source coding theorem says that the Huffman code for a random variable $X$ has an expected length strictly less than $H(X) + 1$. Give an example of a random variable for which the expected length of the Huffman code (without any source extension) is very close to $H(X) + 1$.

**Problem 7.** A channel encoder maps blocks of two bits to five-bit (channel) codewords. The four possible codewords are 00000, 01000, 10001, and 11111. A codeword is transmitted over the BSC with crossover probability $p = 0.1$. What is the minimum (Hamming) distance $d_{min}$ among the codewords?

**Problem 8.** Consider random variables $X$ and $Y$ whose joint pmf is given by

$$p_{X,Y}(x,y) = \begin{cases} c(x+y), & x \in \{1,3\} \text{ and } y \in \{2,4\}, \\ 0, & \text{otherwise.} \end{cases}$$

Evaluate the following quantities.

(a) $c$

(b) $H(X,Y)$

(c) $H(X)$

(d) $H(Y)$

(e) $H(X|Y)$

(f) $H(Y|X)$

(g) $I(X;Y)$