EES 351: In-Class Exercise # 9

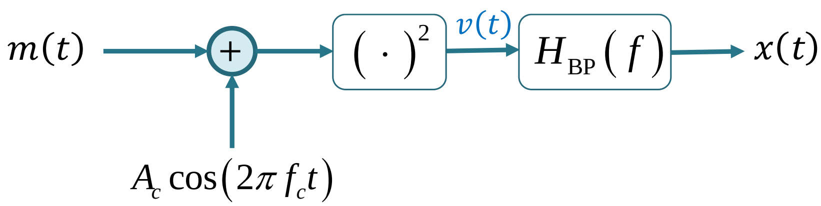
Date: 23 / 9 / 2020			
Name	ID (last 3 digits)		

Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **[ENRE] Explanation is not required for this exercise.**
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member.
     (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

1. Consider the "modulator" shown below. Note that the first operation is a summation (not multiplication).



$\boxed{(\cdot)^2}$ is a "square" device; its output is created by squaring its input in the **time** domain.

$\boxed{H_{BP}(f)}$ is an LTI device whose **frequency response** is

$$H_{BP}(f)=\begin{cases}1, & |f-f_c|\le 332,\\ 1, & |f+f_c|\le 332,\\ 0, & \text{otherwise.}\end{cases}$$

Let $A_c = 2$, $f_c = 2018$, and

$$M(f)=\begin{cases}1, & |f|\le 54,\\ 0, & \text{otherwise.}\end{cases}$$

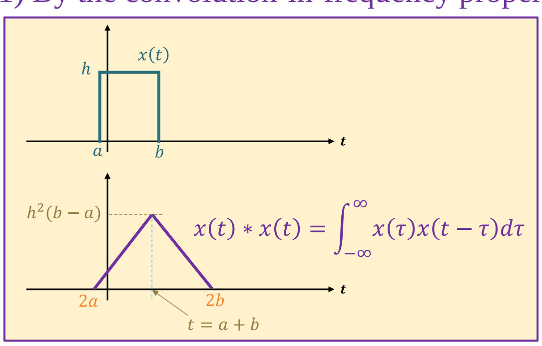
a. Plot the corresponding $X(f)$.

$v(t) = \left(m(t) + 2\cos(2\pi(2018)t)\right)^2 = m^2(t) + 4\cos^2(2\pi(2018)t) + 4m(t)\cos(2\pi(2018)t)$.

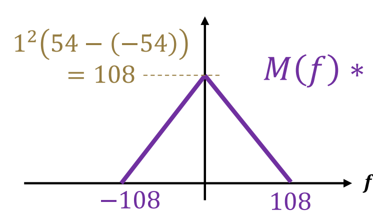
$v(t)$ is input into a band-pass filter. So, we should look at $V(f)$.

There are three terms in $v(t)$. Let's analyze them in the frequency domain separately.

(1) By the convolution-in-frequency property: $m^2(t) = m(t)\times m(t) \overset{\mathcal{F}}{\rightleftharpoons} M(f) * M(f)$.



In lecture, we have seen how to do self-convolution of a rectangular function. In this problem, $a = -54$, $b = 54$, $h = 1$, and the dummy variable is f instead of t. This gives



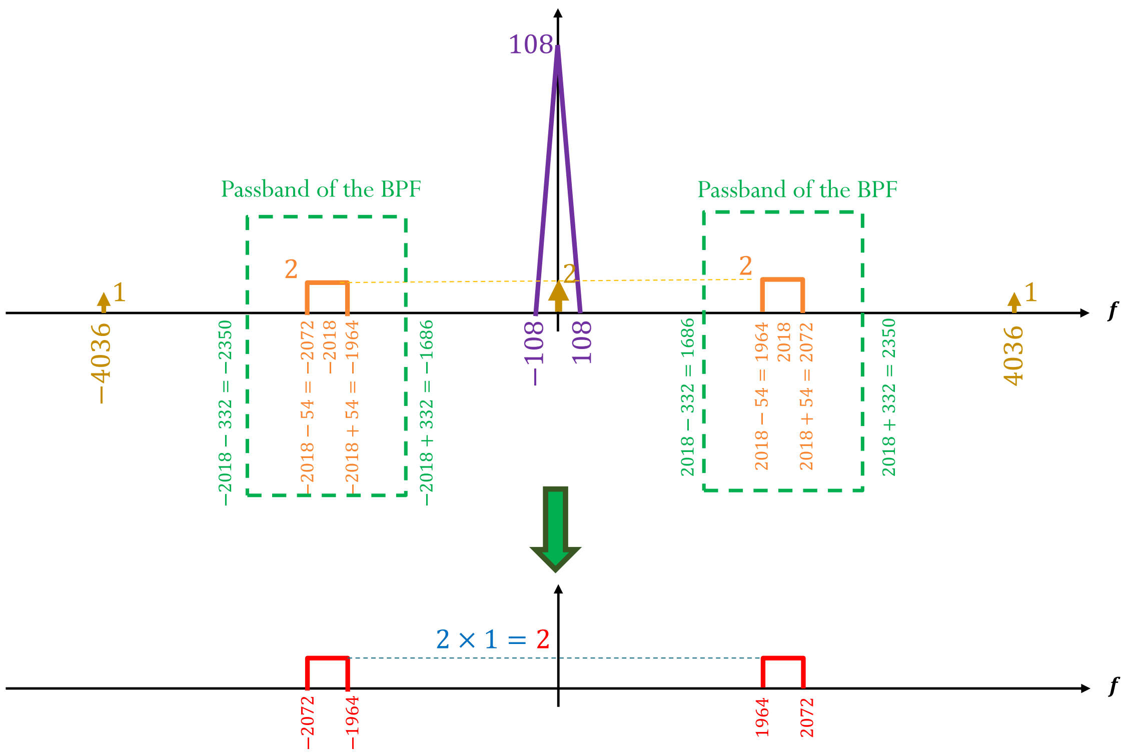
(Actually, the actual shape of the convolution is not as important as the fact that the result is band-limited to 128 Hz.)

(2) From $\cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$, we have

$$4\cos^2(2\pi(2018)t) = 2 + 2\cos(2\pi(4036)t).$$

(3) Fourier transform of the term $4m(t)\cos(2\pi(2018)t)$ can be found by our modulation formula:

$$g(t)A\cos(2\pi(f_c)t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{A}{2}G(f-f_c) + \frac{A}{2}G(f-(-f_c)).$$



b. (Optional) Find $m(t)$ and $x(t)$.

$M(f)$ is a rectangular function in the frequency domain. So, $m(t)$ is a sinc function in the time domain. The area of the rectangular function is $1 \times 54 - (-54) = 108$. So,

$$m(t) = 108\,\mathrm{sinc}(2\pi f_m t),$$

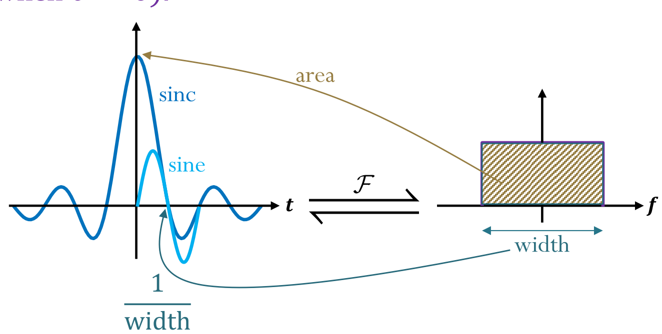
for some f_m.

The width of the rectangular function is $54 - (-54) = 108$. So, the first zero-crossing of the sinc function should occur at $t = \frac{1}{108}$. From $\mathrm{sinc}(\theta) = \frac{\sin(\theta)}{\theta}$, we see that the zero-crossing of $\mathrm{sinc}(\theta)$ are the same as the zero-crossings of $\sin(\theta)$ (except when $\theta = 0$).

Period of sine $= 2 \times \frac{1}{108}$.

Frequency of sine $= \frac{1}{\text{Period of sine}} = \frac{108}{2}$

So, $f_m = \frac{108}{2}$ and $m(t) = 108\,\mathrm{sinc}(108\pi t)$



Back in the analysis of $v(t)$, we saw that $4m(t)\cos(2\pi(2018)t)$ passes through the filter unchanged. So, $x(t) = 4m(t)\cos(2\pi(2018)t) = 432\,\mathrm{sinc}(128\pi t)\cos(4026\pi t)$.