EES 351: In-Class Exercise # 7

Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

Date: 16 / 9 / 2020			
Name	ID (last 3 digits)		

1. Consider a channel with multipath propagation. Its impulse response is of the form

$$h(t)=\sum_{k=1}^{\nu}\beta_k\delta(t-\tau_k).$$

a. Suppose $\nu=2,\ \beta_1=\beta_2=3,\ \tau_1=2,\ \tau_2=5$.

For each of the following channel input $x(t)$, find the corresponding channel output $y(t)$.

Note that the output should be of the form $y(t)=A\cos(2\pi f_0 t+\theta_0)$ for some constants A, f_0, and θ_0 where θ_0 is in degrees.

Channel input	Channel output
$x(t)=\cos(\pi t)$	$y(t)=3x(t-2)+3x(t-5)$ $=3\cos(\pi(t-2))+3\cos(\pi(t-5))$ $=3\cos(\pi t-2\pi)+3\cos(\pi t-5\pi)$ $\cos(\theta)=\cos(\theta+2\pi)$ $\quad$ $\cos(\theta)=\cos(\theta+2\times 2\pi)$ $\quad$ $\cos(\theta-\pi)=-\cos(\theta)$ $=3\cos(\pi t)-3\cos(\pi t)$ $=0\cos(\pi t+0°)\equiv 0$
$x(t)=\cos\left(\frac{\pi}{2}t\right)$	$y(t)=3x(t-2)+3x(t-5)$ $=3\cos\left(\frac{\pi}{2}(t-2)\right)+3\cos\left(\frac{\pi}{2}(t-5)\right)$ $=3\cos\left(\frac{\pi}{2}t-\pi\right)+3\cos\left(\frac{\pi}{2}t-\frac{5\pi}{2}\right)$ $\cos(\theta)=\cos(\theta+2\pi)$ Conversion to phasor form $\Leftrightarrow 3\angle -180°+3\angle -90°=3\sqrt{2}\angle -135°$ Conversion back to time domain $\Leftrightarrow 3\sqrt{2}\cos\left(\frac{\pi}{2}t-135°\right)$

2π period of cosine: $\cos(\theta)=\cos(\theta+n2\pi)$ for any integer n.

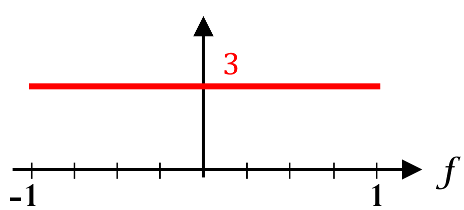
Because the amplitude is 0, any angle and freq. are OK here.

We can use phasor representation to combine sinusoids with the same frequency.

Note that the $2\pi f_0 t$ part of the cosine should be the same. Here, it is $\frac{\pi}{2}t$.

b. Suppose $\nu=1,\ \beta_1=3,\ \tau_1=2$.

Plot $|H(f)|$ from $f=-1$ to $f=1$ Hz.



When $\nu=1$, we have $h(t)=\beta_1\delta(t-\tau_1)$. With the provided values, we have

$$h(t)=3\delta(t-2).$$

Therefore, $H(f)=3e^{-j2\pi(2)f}$ and $|H(f)|=3\left|e^{-j4\pi f}\right|\equiv 3\times 1=3$.

Note that this is a distortionless channel. So, the magnitude spectrum should be flat.