EES 351: In-Class Exercise # 6

Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
      - PDF only.
      - Only for those who can directly work on the posted files using devices with pen input.
      - Paper size should be the same as the posted file.
      - No scanned work, photos, or screen capture.
      - Your file name should start with the 10-digit student ID of one member.
        (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

Date: 11 / 9 / 2020			
Name	ID (last 3 digits)		

In this problem, we have three "devices".

- $\boxed{(\cdot)^2}$ is a "square" device. As the name suggests, its output is created by squaring its input in the **time** domain.
- $\boxed{H_1(f)}$ is an LTI device whose **frequency response** is $H_1(f) = \begin{cases} 1, & |f| < 315, \\ 0, & \text{otherwise.} \end{cases}$
- $\boxed{H_2(f)}$ is an LTI device whose **frequency response** is $H_2(f) = \begin{cases} 1, & |f| > 315, \\ 0, & \text{otherwise.} \end{cases}$

Find the output $y(t)$ for each of the systems below.

★ Recall that

$$e^{j2\pi f_0 t} \rightarrow \boxed{H(f)} \rightarrow H(f_0)e^{j2\pi f_0 t}$$

$$j2\pi f_0 t = 351\pi t \Rightarrow f_0 = \frac{351}{2} = 175.5$$

(a) $x(t) = e^{j351\pi t} \longrightarrow \boxed{H_1(f)} \longrightarrow y(t)$

$H_1(175.5) = 1$ because $|175.5| < 315$.

$$y(t) = H_1(f_0)e^{j2\pi f_0 t} = H_1(175.5)e^{j2\pi(175.5)t} = 1e^{j351\pi t} = e^{j351\pi t}$$

✧ Recall that

$$\cos(2\pi f_0 t) \rightarrow \boxed{H(f)} \rightarrow \frac{1}{2}H(f_0)e^{j2\pi f_0 t} + \frac{1}{2}H(-f_0)e^{-j2\pi f_0 t}$$

$$= H(f_0)\cos(2\pi f_0 t)$$

when $H(f)$ is an even function which is the case here

(b) $x(t) = \cos(351\pi t) \longrightarrow \boxed{H_1(f)} \longrightarrow y(t)$

$$y(t) = H_1(f_0)\cos(2\pi f_0 t) = \underbrace{H_1(175.5)}_{1}\cos(2\pi(175.5)t) = \cos(351\pi t)$$

(c) $x(t) = \cos(351\pi t) \longrightarrow \boxed{H_2(f)} \longrightarrow y(t)$

$$y(t) = H_2(f_0)\cos(2\pi f_0 t) = \underbrace{H_2(166.5)}_{0}\cos(2\pi(166.5)t) = 0$$

$H_2(175.5) = 0$ because $|175.5| \ngtr 315$.

(d) $x(t) = \cos(351\pi t) \longrightarrow \boxed{(\cdot)^2} \xrightarrow{x^2(t)} \boxed{H_1(f)} \longrightarrow y(t)$

(e)

$$x^2(t) = \cos^2(351\pi t) = \left(\frac{e^{j351\pi t} + e^{-j351\pi t}}{2}\right)^2 = \frac{1}{4}e^{j2\pi(351)t} + \frac{1}{2} + \frac{1}{4}e^{j2\pi(-351)t}$$

One can view the constant $\frac{1}{2}$ as a complex-expo. function

$$\frac{1}{2}e^{j2\pi(0)t}$$

whose freq. is 0

So, $x^2(t)$ is simply a linear combination of complex-exponential functions. Therefore, we can apply our ★ to each term.

$$y(t) = \frac{1}{4}\underbrace{H_1(351)}_{0}e^{j2\pi(351)t} + \frac{1}{2}\overbrace{H_1(0)}^{1} + \frac{1}{4}\underbrace{H_1(-351)}_{0}e^{j2\pi(-351)t} = \frac{1}{2}$$