EES 351: In-Class Exercise # 1 - Sol

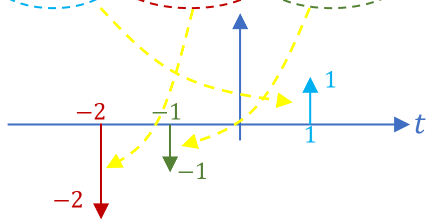
Date: 21 / 8 / 2020			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

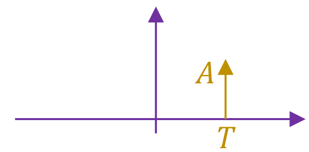
1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

1) Plot $\delta(t-1)-2\delta(t+2)-\delta(t+1)$.



This is the plot of $A\delta(t-T)$:



2) Evaluate the following integrals:

a) $\int_{-\pi}^{\pi}\delta(t)dt = 1$ because $0 \in (-\pi,\pi)$.

Remark: As discussed in class, writing the limits of integration in this form is ambiguous when dealing with δ-function because we don't know whether the boundary points are included in the integration or not. However, this does not affect our answer in this part because 0 will still be within the interval of integration even without the boundary points.

$$\int_A \delta(t)\,dt = \begin{cases}1, & 0\in A\\ 0, & 0\notin A\end{cases}$$

b) $\int_{\pi}^{2\pi}\delta(t-3)dt = 0$

$\left.\begin{array}{l} c=3\\ A=[\pi,2\pi]\\ g(t)\equiv 1\end{array}\right\} c\notin A$

Remark: As discussed in class, writing the limits of integration in this form is ambiguous when dealing with δ-function because we don't know whether the boundary points are included in the integration or not. However, this does not affect our answer in this part because c will still be outside A even when we also include the two boundary points.

(Extended) **Sifting Property**:

$$\int_A g(t)\delta(t-c)\,dt = \begin{cases}g(c), & c\in A\\ 0, & c\notin A\end{cases}$$

c) $\int_{\pi}^{2\pi}e^{j\frac{\pi}{2}t}\delta(t)dt = 0$

$\left.\begin{array}{l} c=0\\ A=[\pi,2\pi]\end{array}\right\} c\notin A$

$g(t)=e^{j\frac{\pi}{2}t}$

Similar remark to part (b)

d) $\int_{0}^{2\pi}e^{j\frac{\pi}{2}t}\delta(t-3)dt = g(c) = e^{j\frac{\pi}{2}3} = -j$

$\left.\begin{array}{l} c=3\\ A=(0,2\pi)\end{array}\right\} c\in A$

$g(t)=e^{j\frac{\pi}{2}t}$

Similar remark to part (a)

e) $\int_{0}^{\infty}\delta(t-1)-2\delta(t+2)-\delta(t+1)dt = \int_0^\infty \delta(t-1)\,dt - 2\int_0^\infty \delta(t+2)\,dt - \int_0^\infty \delta(t+1)\,dt$

$1\in(0,\infty)$ $\quad -2\notin[0,\infty)$ $\quad -1\notin[0,\infty)$

$= 1 \quad +0 \quad +0$

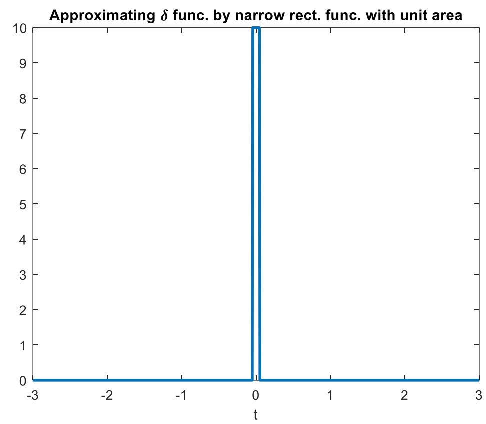
$= 1$

f) (optional) $\int_{-\infty}^{\infty}\delta(t^2-2t)dt$

In the optional question, we would like to calculate $\int_{-\infty}^{\infty} \delta\left(g\left(t\right)\right) dt$ where $g\left(t\right)=t^2-2t$.

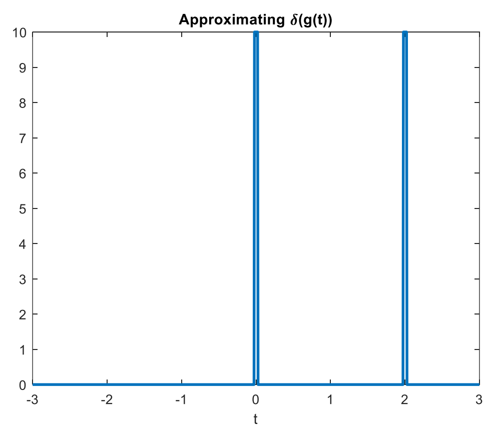
Here are some hints.

Let's look at the limiting approximation of $\delta\left(t\right)$. Consider a rectangular function centered at origin whose width is ε. To be a delta function, we need the area = 1; therefore, the height must be $\frac{1}{\varepsilon}$. In the MATLAB plot below, we use $\varepsilon=0.1$. As expected, there is a "spike" at $t=0$.




```
close all; clear all;
ep = 1e-1;
t = linspace(-3,3,1e3);

% The rectangular func. that approximates the delta func.
d = 1/ep * rectangularPulse(-ep/2,ep/2,t);
plot(t,d,'LineWidth',2)
title('Approximating \delta func. by narrow rect. func. with unit area')
xlabel('t')

g = t.*(t-2);

figure
plot(t,g,'LineWidth',2)
ylim([-3,3])
grid on
title('g(t)')
xlabel('t')

figure
% plugging g(t) into the delta function
deltag = 1/ep * rectangularPulse(-ep/2,ep/2,g);
plot(t,deltag,'LineWidth',2)
title('Approximating \delta(g(t))')
xlabel('t')
```

Now, let's try to plot $\delta\left(g\left(t\right)\right)$ where $g\left(t\right)=t^2-2t$.



Note that there are two "spikes" at $t=0$ and $t=2$. This is expected because we know that the spikes will show up when the argument of $\delta\left(\cdot\right)$ is 0. Here, $g\left(t\right)=t^2-2t$ is zero at $t=0$ and $t=2$.

The area under the graph above approximates $\int_{-\infty}^{\infty} \delta\left(g\left(t\right)\right) dt$. This area is the sum of the areas under the two spikes. Note, however, that the area under each spike is not one anymore; the spikes seem to be narrower. How can we find their areas? (Furthermore, can we eliminate MATLAB from this calculation?)