EES 351: In-Class Exercise # 1 - Sol

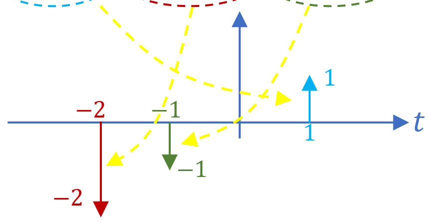
Date: 21 / 8 / 2020			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

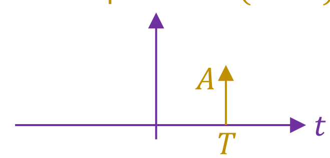
1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member.
     (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

1) Plot $\delta(t-1) - 2\delta(t+2) - \delta(t+1)$.



This is the plot of $A\delta(t-T)$:



2) Evaluate the following integrals:

a) $\int_{-\pi}^{\pi} \delta(t)dt = 1$ because $0 \in (-\pi, \pi)$.

Remark: As discussed in class, writing the limits of integration in this form is ambiguous when dealing with δ-function because we don't know whether the boundary points are included in the integration or not. However, this does not affect our answer in this part because 0 will still be within the interval of integration even without the boundary points.

$$\int_A \delta(t)\, dt = \begin{cases} 1, & 0 \in A \\ 0, & 0 \notin A \end{cases}$$

b) $\int_{\pi}^{2\pi} \delta(t-3)dt = 0$

$$\left.\begin{array}{l} c = 3 \\ A = [\pi, 2\pi] \end{array}\right\} c \notin A$$
$$g(t) \equiv 1$$

Remark: As discussed in class, writing the limits of integration in this form is ambiguous when dealing with δ-function because we don't know whether the boundary points are included in the integration or not. However, this does not affect our answer in this part because c will still be outside A even when we also include the two boundary points.

(Extended) **Sifting Property**:

$$\int_A g(t)\delta(t-c)\, dt = \begin{cases} g(c), & c \in A \\ 0, & c \notin A \end{cases}$$

c) $\int_{\pi}^{2\pi} e^{j\frac{\pi}{2}t} \delta(t)dt = 0$

$$\left.\begin{array}{l} c = 0 \\ A = [\pi, 2\pi] \end{array}\right\} c \notin A$$
$$g(t) = e^{j\frac{\pi}{2}t}$$

Similar remark to part (b)

d) $\int_{0}^{2\pi} e^{j\frac{\pi}{2}t} \delta(t-3)dt = g(c) = e^{j\frac{\pi}{2}3} = -j$

$$\left.\begin{array}{l} c = 3 \\ A = (0, 2\pi) \end{array}\right\} c \in A$$
$$g(t) = e^{j\frac{\pi}{2}t}$$

Similar remark to part (a)

e) $\int_{0}^{\infty} \delta(t-1) - 2\delta(t+2) - \delta(t+1)dt = \int_0^\infty \delta(t-1)\, dt - 2\int_0^\infty \delta(t+2)\, dt - \int_0^\infty \delta(t+1)\, dt$

$1 \in (0, \infty)$ $\quad$ $-2 \notin [0, \infty)$ $\quad$ $-1 \notin [0, \infty)$

$= 1 \quad +0 \quad +0$

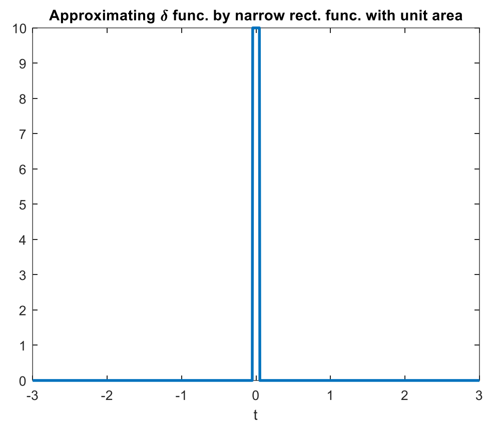
$= 1$

f) (optional) $\int_{-\infty}^{\infty} \delta(t^2 - 2t)dt$

In the optional question, we would like to calculate $\int_{-\infty}^{\infty} \delta\left(g\left(t\right)\right)dt$ where $g\left(t\right)=t^2-2t$.

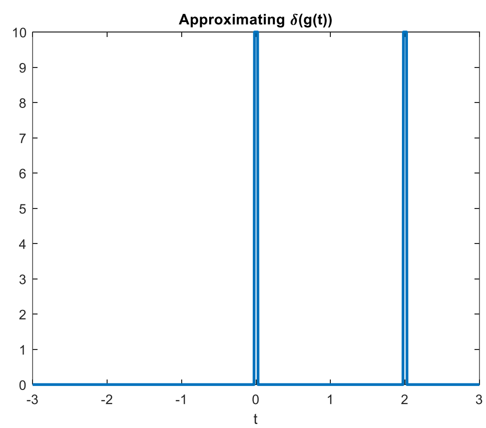
Here are some hints.

Let's look at the limiting approximation of $\delta\left(t\right)$. Consider a rectangular function centered at origin whose width is ε. To be a delta function, we need the area = 1; therefore, the height must be $\frac{1}{\varepsilon}$. In the MATLAB plot below, we use $\varepsilon=0.1$. As expected, there is a "spike" at $t=0$.




```
close all; clear all;
ep = 1e-1;
t = linspace(-3,3,1e3);

% The rectangular func. that approximates the delta func.
d = 1/ep * rectangularPulse(-ep/2,ep/2,t);
plot(t,d,'LineWidth',2)
title('Approximating \delta func. by narrow rect. func. with unit area')
xlabel('t')

g = t.*(t-2);

figure
plot(t,g,'LineWidth',2)
ylim([-3,3])
grid on
title('g(t)')
xlabel('t')

figure
% plugging g(t) into the delta function
deltag = 1/ep * rectangularPulse(-ep/2,ep/2,g);
plot(t,deltag,'LineWidth',2)
title('Approximating \delta(g(t))')
xlabel('t')
```

Now, let's try to plot $\delta\left(g\left(t\right)\right)$ where $g\left(t\right)=t^2-2t$.



Note that there are two "spikes" at $t=0$ and $t=2$. This is expected because we know that the spikes will show up when the argument of $\delta\left(\cdot\right)$ is 0. Here, $g\left(t\right)=t^2-2t$ is zero at $t=0$ and $t=2$.

The area under the graph above approximates $\int_{-\infty}^{\infty} \delta\left(g\left(t\right)\right)dt$. This area is the sum of the areas under the two spikes. Note, however, that the area under each spike is not one anymore; the spikes seem to be narrower. How can we find their areas? (Furthermore, can we eliminate MATLAB from this calculation?)

EES 351: In-Class Exercise # 2

Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. [ENRE] = Explanation is not required for this exercise.
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

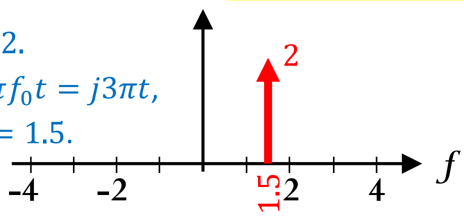
Date: 26 / 8 / 2020			
Name	ID (last 3 digits)		
Prapun	5	5	5

1. [ENRPr] Consider each $g(t)$ defined below.
   Let $G(f)$ be its Fourier transform. Plot $G(f)$ from $f=-4$ to $f=4$ Hz.

a. $g(t)=2e^{j3\pi t}$

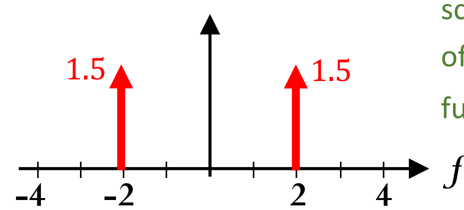
$Ae^{j2\pi f_0 t} \xrightleftharpoons{\mathcal{F}} A\delta(f-f_0)$

Here, $A=2$.
Setting $j2\pi f_0 t = j3\pi t$,
we get $f_0 = 1.5$.



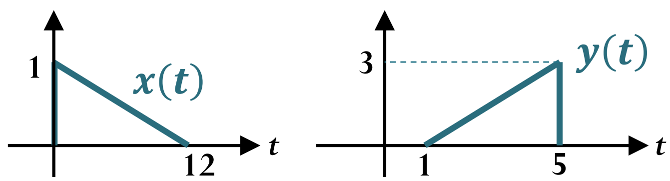
b. $g(t)=3\cos(4\pi t)$

Don't forget to scale the size of each δ-function by $\frac{1}{2}$.



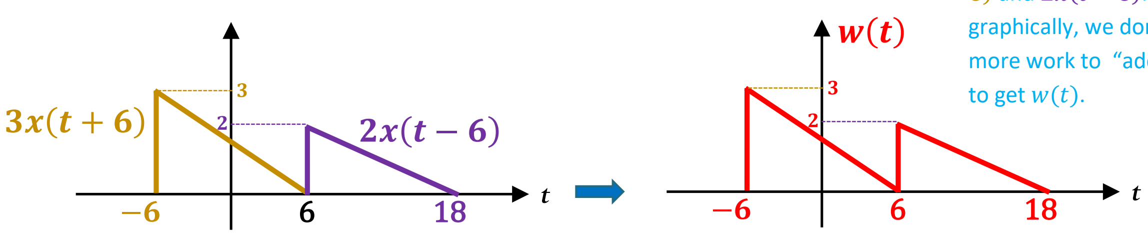
$A\cos(2\pi f_0 t) \xrightleftharpoons{\mathcal{F}} \frac{A}{2}\delta(f-(-f_0))+\frac{A}{2}\delta(f-f_0)$

2. [ENRPr] Signals $x(t)$ and $y(t)$ are plotted below.



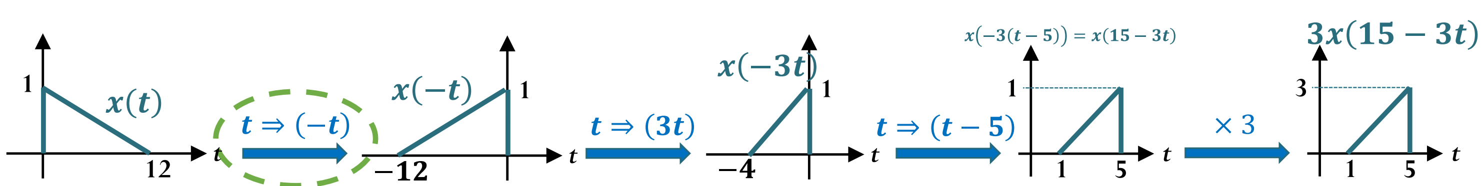
a) Plot the signal $w(t)=2x(t-6)+3x(t+6)$.

Remark: There is no nonzero overlapping part between $3x(t+6)$ and $2x(t-6)$. Therefore, graphically, we don't need to do any more work to "add" the two graphs to get $w(t)$.



b) Suppose $y(t)=c_1 x(c_2 t + c_3)$. Find the values of the constants $c_1, c_2,$ and c_3.

$c_1 =$ 3 , $c_2 =$ −3 , $c_3 =$ 15 .



This notation means we replace t by "$-t$".

Caution: One common mistake is that, in the third step, when we shift the graph to the right by 5 units, we can't just put "−5" blindly into the expression and get $x(-3t-5)$; we need to replace t by $t-5$.

EES 351: In-Class Exercise # 3 - Sol

Instructions

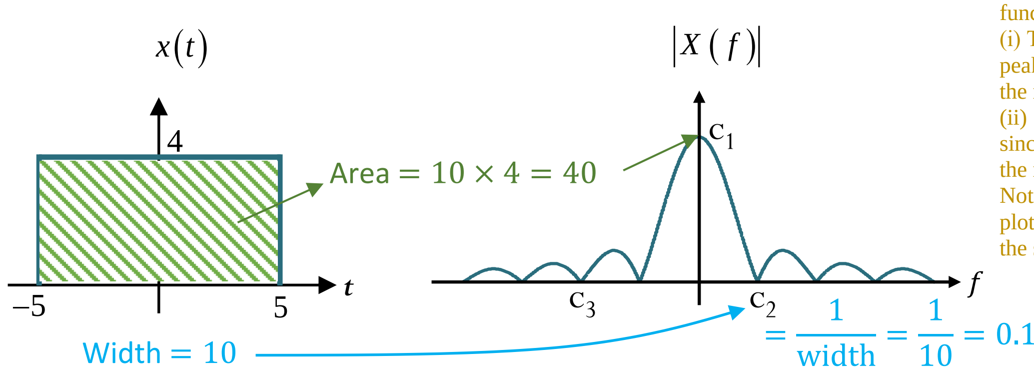
1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. [ENRE] = Explanation is not required for this exercise.
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

Date: 28 / 8 / 2020			
Name	ID (last 3 digits)		
Prapun	5	5	5

For this question, we apply observation discussed in the lecture notes [2.13]:
(0) The Fourier transform of a rectangular function is a sinc function.
(i) The height of the sinc function's peak is the same as the area under the rectangular function.
(ii) The first zero crossing of the sinc function occurs at 1/(width of the rectangular function).
Note also that in the magnitude plot, the negative-valued parts of the sinc function are flipped up.

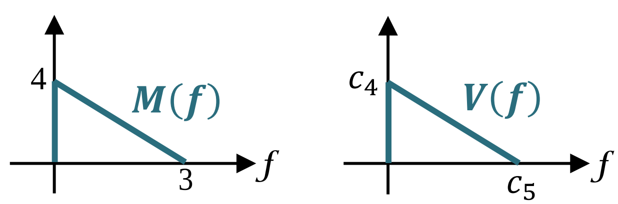
1. A signal and its magnitude spectrum are plotted below.



Find the values of the constants (corresponding to some zeroes and the peak value) shown in the plots.

$$c_1 = \underline{40}, c_2 = \underline{0.1}, c_3 = \underline{-2c_2} = -0.2$$

2. Consider a signal $m(t)$ and another signal $v(t) = m(5t)$.
   Their corresponding Fourier transforms are plotted below.



Caution: The relationship between the two signals is given in the time domain. However, the plots are given in the frequency domain.

Find the values of the constants in the plot of $V(f)$:

$$c_4 = \underline{\tfrac{4}{5} = 0.8}, c_5 = \underline{15}$$

For $v(t) = m(5t)$, by the scale-change theorem [2.32 eq. (21)], we have

$$V(f) = \frac{1}{|5|} M\left(\frac{f}{5}\right) = \frac{1}{5} M\left(\frac{f}{5}\right).$$

In the previous exercise, we worked on time manipulation. Note that, back then, "time" was our dummy variable. Here, it's the frequency f. We can get $M\left(\frac{f}{5}\right)$ from $M(f)$ by replacing f by $\frac{f}{5}$; therefore, graphically, this is a horizontal expansion by a factor of 5. This implies $c_5 = 5 \times 3 = 15$.

Finally, the $\frac{1}{5}$ in the front simply scales the height of graph by a factor of $\frac{1}{5}$. This implies $c_4 = \frac{1}{5} \times 4 = \frac{4}{5} = 0.8$.

EES 351: In-Class Exercise # 4

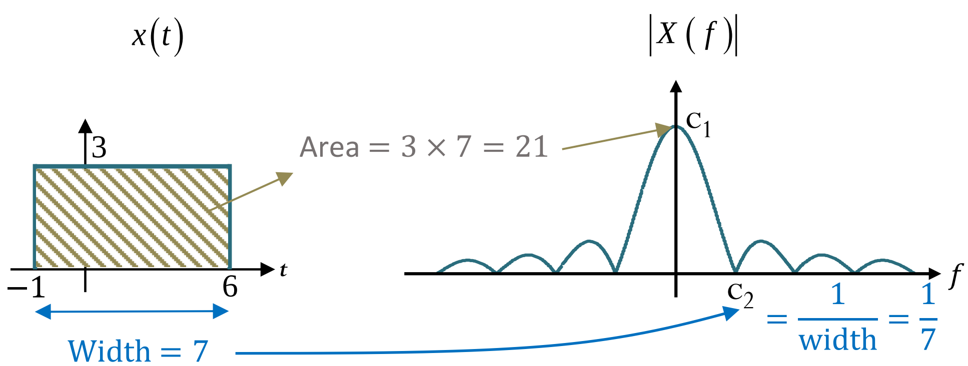
Date: 2 / 9 / 2020			
Name	ID (last 3 digits)		

Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. [ENRE] = Explanation is not required for this exercise.
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

1. A signal and its magnitude spectrum are plotted below.

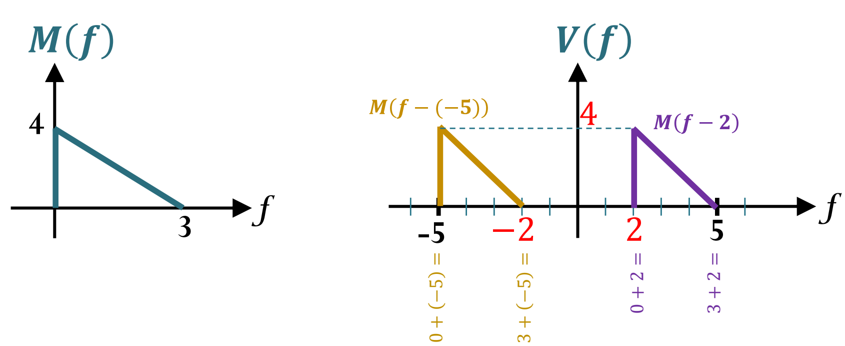


Find the values of the constants (corresponding to some zeroes and the peak value) shown in the plots.

$$c_1 = \underline{\;21\;},\; c_2 = \underline{\;\tfrac{1}{7}\;}.$$

This problem is similar to the one we have worked on in the previous exercise. However, the rectangular function is not centered at $t = 0$; it is time-shifted. From the time-shift property (2.31) of Fourier transform, we know that the magnitude spectrum plot won't be affected by this time-shifting. So, we can still use 2.13. In particular,
(0) The Fourier transform of a rectangular function is a sinc function.
(i) The height of the sinc function's peak is the same as the area under the rectangular function.
(ii) The first zero crossing of the sinc function occurs at 1/(width of the rectangular function).

2. Consider a signal $m(t)$. Its Fourier transform $M(f)$ is plotted below.
   Let $v(t) = e^{-j10\pi t}m(t) + e^{j4\pi t}m(t)$. $= e^{j2\pi(-5)t}m(t) + e^{j2\pi(2)t}m(t)$
   Plot $V(f)$ in the corresponding space below.



Note that the plots of $M(f-(-5))$ and $M(f-2)$ do not have any (nonzero) overlapping part, so the plot of their sum is the same as what we already have here.

From the frequency-shift property of Fourier transform, we know that $e^{j2\pi f_0 t}m(t) \xrightarrow{\mathcal{F}} M(f-f_0)$.

In particular, $e^{j2\pi(-5)t}m(t) \xrightarrow{\mathcal{F}} M(f-(-5))$ and

$e^{j2\pi(2)t}m(t) \xrightarrow{\mathcal{F}} M(f-2)$.

Therefore, $e^{j2\pi(-5)t}m(t) + e^{j2\pi(2)t}m(t) \xrightarrow{\mathcal{F}} M(f-(-5)) + M(f-2)$.

EES 351: In-Class Exercise # 5

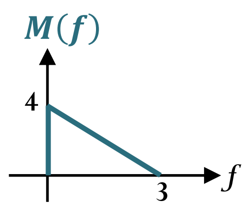
Date: 9 / 9 / 2020			
Name	ID (last 3 digits)		

Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. [ENRE] Explanation is not required for this exercise.
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

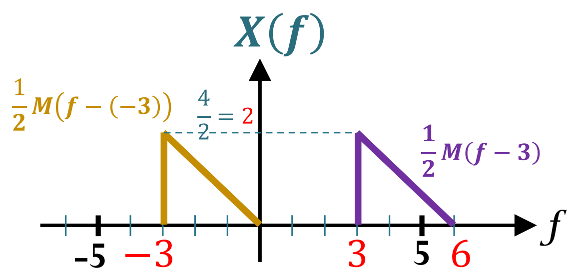
1. Consider a signal $m(t)$. Its Fourier transform $M(f)$ is plotted below.



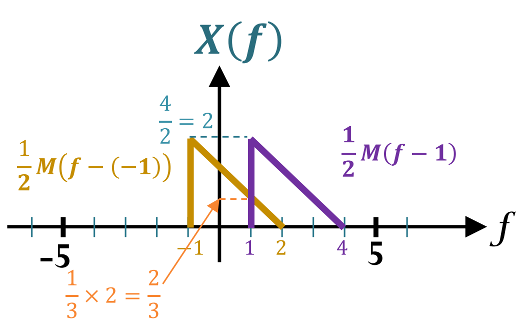
Recall that

$$g(t)\cos(2\pi(f_c)t) \overset{F}{\rightleftharpoons} \frac{1}{2}G(f-f_c)+\frac{1}{2}G(f-(-f_c)).$$

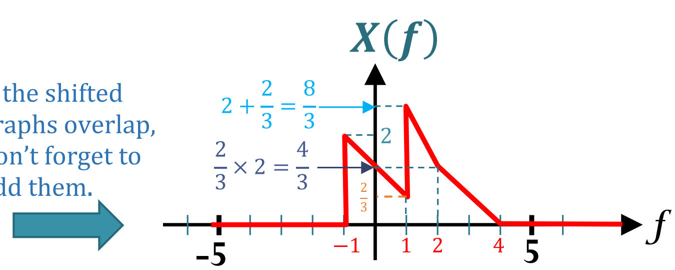
a. Let $x(t)=\cos(6\pi t)m(t)$.
   Plot $X(f)$ in the corresponding space below.

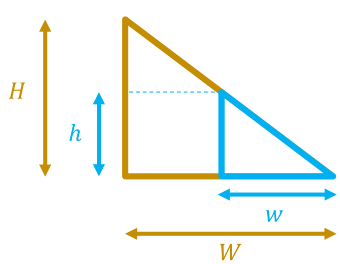


b. Let $x(t)=\cos(2\pi t)m(t)$.
   Plot $X(f)$ in the corresponding space below.



If the shifted graphs overlap, don't forget to add them.





Similar Triangles:

$$\frac{h}{H}=\frac{w}{W}$$

EES 351: In-Class Exercise # 6

Date: 11 / 9 / 2020			
Name	ID (last 3 digits)		

Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member.
   (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

In this problem, we have three "devices".

- $\boxed{(\cdot)^2}$ is a "square" device. As the name suggests, its output is created by squaring its input in the **time** domain.
- $\boxed{H_1(f)}$ is an LTI device whose **frequency response** is $H_1(f) = \begin{cases} 1, & |f| < 315, \\ 0, & \text{otherwise.} \end{cases}$
- $\boxed{H_2(f)}$ is an LTI device whose **frequency response** is $H_2(f) = \begin{cases} 1, & |f| > 315, \\ 0, & \text{otherwise.} \end{cases}$

✶ Recall that

$$e^{j2\pi f_0 t} \to \boxed{H(f)} \to H(f_0)e^{j2\pi f_0 t}$$

Find the output $y(t)$ for each of the systems below.

$j2\pi f_0 t = 351\pi t \Rightarrow f_0 = \frac{351}{2} = 175.5$

(a) $x(t) = e^{j351\pi t} \longrightarrow \boxed{H_1(f)} \longrightarrow y(t)$

$H_1(175.5) = 1$ because $|175.5| < 315$.

$$y(t) = H_1(f_0)e^{j2\pi f_0 t} = H_1(175.5)e^{j2\pi(175.5)t} = 1e^{j351\pi t} = e^{j351\pi t}$$

✧ Recall that

$$\cos(2\pi f_0 t) \to \boxed{H(f)} \to \frac{1}{2}H(f_0)e^{j2\pi f_0 t} + \frac{1}{2}H(-f_0)e^{-j2\pi f_0 t}$$

$$= H(f_0)\cos(2\pi f_0 t)$$

when $H(f)$ is an even function which is the case here

(b) $x(t) = \cos(351\pi t) \longrightarrow \boxed{H_1(f)} \longrightarrow y(t)$

$$y(t) = H_1(f_0)\cos(2\pi f_0 t) = \overbrace{H_1(175.5)}^{1}\cos(2\pi(175.5)t) = \cos(351\pi t)$$

(c) $x(t) = \cos(351\pi t) \longrightarrow \boxed{H_2(f)} \longrightarrow y(t)$

$$y(t) = H_2(f_0)\cos(2\pi f_0 t) = \overbrace{H_2(175.5)}^{0}\cos(2\pi(175.5)t) = 0$$

$H_2(175.5) = 0$ because $|175.5| \ngtr 315$.

(d) $x(t) = \cos(351\pi t) \longrightarrow \boxed{(\cdot)^2} \xrightarrow{x^2(t)} \boxed{H_1(f)} \longrightarrow y(t)$

(e)

$$x^2(t) = \cos^2(351\pi t) = \left(\frac{e^{j351\pi t} + e^{-j351\pi t}}{2}\right)^2 = \frac{1}{4}e^{j2\pi(351)t} + \frac{1}{2} + \frac{1}{4}e^{j2\pi(-351)t}$$

One can view the constant $\frac{1}{2}$ as a complex-expo. function $\frac{1}{2}e^{j2\pi(0)t}$ whose freq. is 0

So, $x^2(t)$ is simply a linear combination of complex-exponential functions. Therefore, we can apply our ✶ to each term.

$$y(t) = \frac{1}{4}\underbrace{H_1(351)}_{0}e^{j2\pi(351)t} + \frac{1}{2}\overbrace{H_1(0)}^{1} + \frac{1}{4}\underbrace{H_1(-351)}_{0}e^{j2\pi(-351)t} = \frac{1}{2}$$

EES 351: In-Class Exercise # 7

Date: 16 / 9 / 2020			
Name	ID (last 3 digits)		

Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

1. Consider a channel with multipath propagation. Its impulse response is of the form

$$h(t)=\sum_{k=1}^{\nu}\beta_k\delta(t-\tau_k).$$

a. Suppose $\nu=2,\ \beta_1=\beta_2=3,\ \tau_1=2,\ \tau_2=5$.
For each of the following channel input $x(t)$, find the corresponding channel output $y(t)$.
Note that the output should be of the form $y(t)=A\cos(2\pi f_0 t+\theta_0)$ for some constants A, f_0, and θ_0 where θ_0 is in degrees.

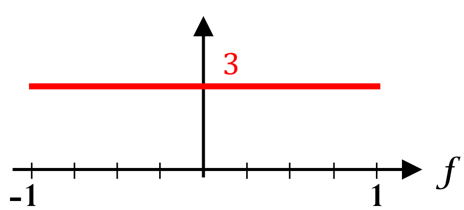
Channel input	Channel output
$x(t)=\cos(\pi t)$	$y(t)=3x(t-2)+3x(t-5)$ $=3\cos(\pi(t-2))+3\cos(\pi(t-5))$ $=3\cos(\pi t-2\pi)+3\cos(\pi t-5\pi)$ $\cos(\theta)=\cos(\theta+2\pi)$ $\cos(\theta)=\cos(\theta+2\times 2\pi)$ $\cos(\theta-\pi)=-\cos(\theta)$ $=3\cos(\pi t)-3\cos(\pi t)$ $=0\cos(\pi t+0°)\equiv 0$ Because the amplitude is 0, any angle and freq. are OK here.
$x(t)=\cos\left(\frac{\pi}{2}t\right)$	$y(t)=3x(t-2)+3x(t-5)$ $=3\cos\left(\frac{\pi}{2}(t-2)\right)+3\cos\left(\frac{\pi}{2}(t-5)\right)$ $=3\cos\left(\frac{\pi}{2}t-\pi\right)+3\cos\left(\frac{\pi}{2}t-\frac{5\pi}{2}\right)$ $\cos(\theta)=\cos(\theta+2\pi)$ Conversion to phasor form $\Leftrightarrow 3\angle-180°+3\angle-90°=3\sqrt{2}\angle-135°$ Conversion back to time domain $\Leftrightarrow 3\sqrt{2}\cos\left(\frac{\pi}{2}t-135°\right)$

2π period of cosine:
$\cos(\theta)=\cos(\theta+n2\pi)$
for any integer n.

We can use phasor representation to combine sinusoids with the same frequency.

Note that the $2\pi f_0 t$ part of the cosine should be the same. Here, it is $\frac{\pi}{2}t$.

b. Suppose $\nu=1,\ \beta_1=3,\ \tau_1=2$.
Plot $|H(f)|$ from $f=-1$ to $f=1$ Hz.



When $\nu=1$, we have $h(t)=\beta_1\delta(t-\tau_1)$. With the provided values, we have

$$h(t)=3\delta(t-2).$$

Therefore, $H(f)=3e^{-j2\pi(2)f}$ and $|H(f)|=3\left|e^{-j4\pi f}\right|\equiv 3\times 1=3$.

Note that this is a distortionless channel. So, the magnitude spectrum should be flat.

EES 351: In-Class Exercise # 8

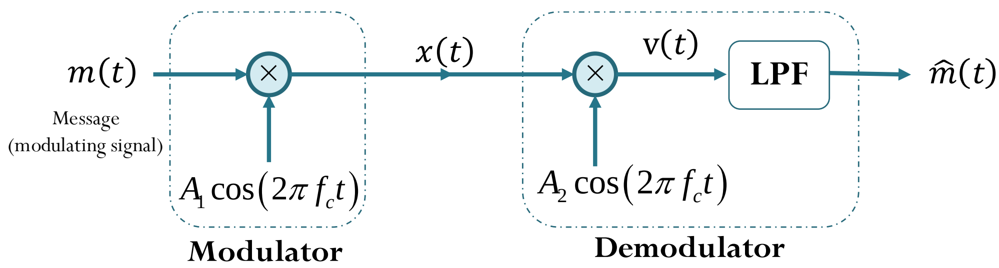
Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **[ENRE] Explanation is not required for this exercise.**
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member.
     (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

Date: 18 / 9 / 2020			
Name	ID (last 3 digits)		

1. Consider a DSB-SC modem with no channel impairment shown below.



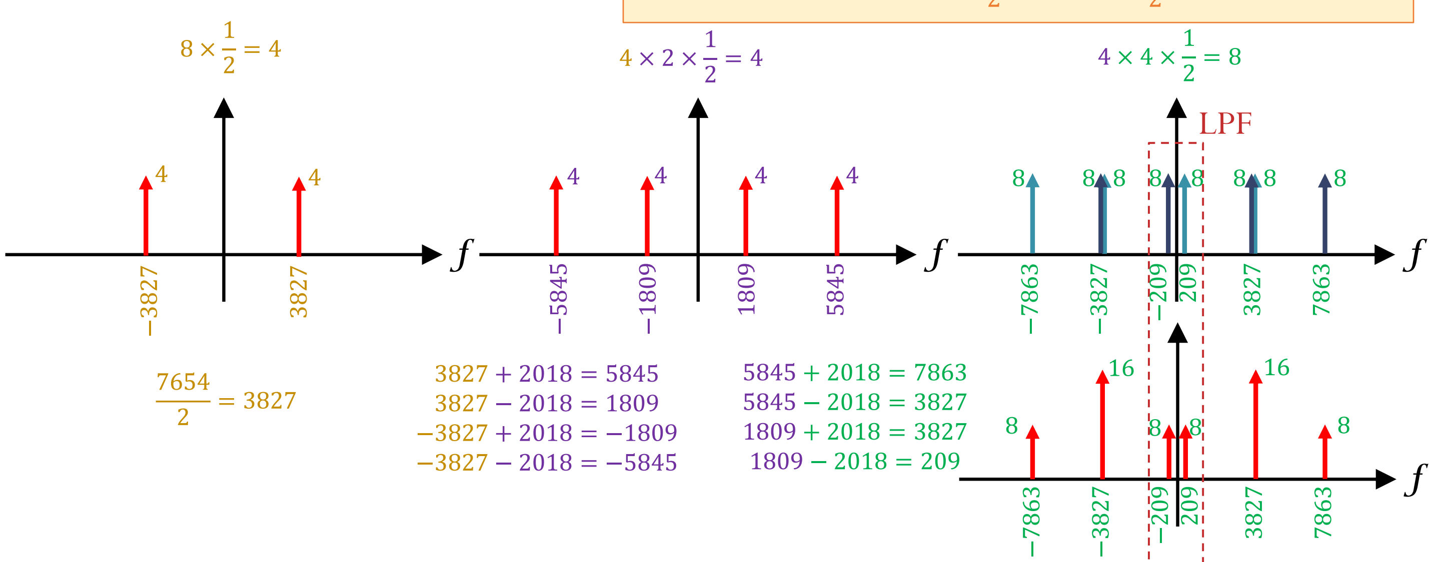
Let $A_1 = 2$, $A_2 = 4$, and $f_c = 2018$ Hz. Suppose LPF has $H_{LP}(f) = \begin{cases} 1, & |f| \le 332, \\ 0, & \text{otherwise.} \end{cases}$

Suppose $m(t) = 8\cos(7654\pi t)$.

a) Plot $M(f), X(f)$, and $V(f)$.

We can try to write down the expression for $x(t)$ and $v(t)$ and then try to find their Fourier transforms. However, here, we not do that. We will work in the frequency domain in all the steps using the modulation property (twice):

$$g(t)A\cos(2\pi(f_c)t) \overset{F}{\rightleftharpoons} \frac{A}{2}G(f - f_c) + \frac{A}{2}G(f - (-f_c))$$



Note that the gain of the LPF in its passband is 1; so there is no further scaling here.

b) Find the corresponding $\hat{m}(t)$.

In the freq. domain, we have $\widehat{M}(f) = H_{LF}(f)V(f)$. Using $V(f)$ from part (a), eliminating the terms at $|f| > 332$, we get

$$\widehat{M}(f) = 8\delta(f - (-209)) + 8\delta(f - 209).$$

Therefore,

$$m(t) = 8e^{j2\pi(-209)t} + 8e^{j2\pi(209)t} = 16\cos(2\pi(209)t) = 16\cos(418\pi t).$$

Alternatively, we can work in the time domain directly.
We will need to use a trigonometric identity that we have proved in our HW:

$$\cos(A)\cos(b) = \frac{1}{2}(\cos(A+B) + \cos(A-B)).$$

Here, we have

$$\begin{aligned} v(t) &= x(t) \times 4\cos(2\pi(2018)t) = (m(t) \times 2\cos(2\pi(2018)t)) \times 4\cos(2\pi(2018)t) \\ &= m(t) \times 8\cos^2(2\pi(2018)t) = m(t) \times 4(1 + \cos(2\pi(2 \times 2018)t)) \\ &= 4m(t) + 4m(t)\cos(2\pi(2 \times 2018)t). \end{aligned}$$

Substituting $m(t) = 8\cos(7654\pi t) = 8\cos(2\pi(3827)t)$, we get

$$\begin{aligned} v(t) &= 32\cos(2\pi(3827)t) + 32\cos(2\pi(3827)t)\cos(2\pi(4036)t) \\ &= 32\cos(2\pi(3827)t) + 16\cos(2\pi(209)t) + 16\cos(2\pi(7863)t). \end{aligned}$$

After the LPF, only sinusoid with freq. not exceeding 332 passes through.
Therefore,

$$m(t) = 16\cos(2\pi(209)t) = 16\cos(418\pi t).$$

EES 351: In-Class Exercise # 9

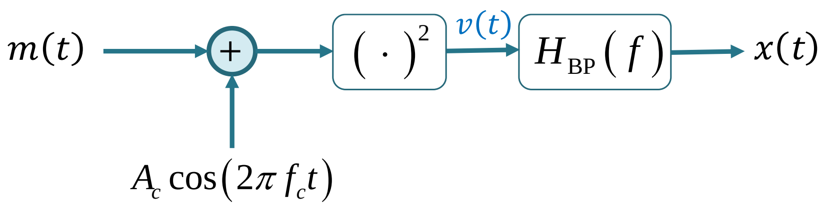
Date: 23 / 9 / 2020			
Name	ID (last 3 digits)		

Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **[ENRE] Explanation is not required for this exercise.**
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member.
     (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

1. Consider the "modulator" shown below. Note that the first operation is a summation (not multiplication).



$\boxed{(\cdot)^2}$ is a "square" device; its output is created by squaring its input in the **time** domain.

$\boxed{H_{BP}(f)}$ is an LTI device whose **frequency response** is

$$H_{BP}(f) = \begin{cases} 1, & |f - f_c| \le 332, \\ 1, & |f + f_c| \le 332, \\ 0, & \text{otherwise.} \end{cases}$$

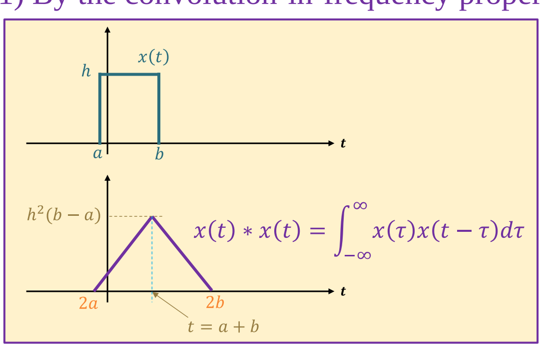
Let $A_c = 2$, $f_c = 2018$, and

$$M(f) = \begin{cases} 1, & |f| \le 54, \\ 0, & \text{otherwise.} \end{cases}$$

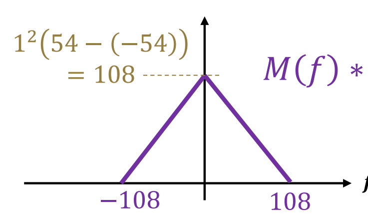
a. Plot the corresponding $X(f)$.

$v(t) = \left(m(t) + 2\cos(2\pi(2018)t)\right)^2 = m^2(t) + 4\cos^2(2\pi(2018)t) + 4m(t)\cos(2\pi(2018)t)$.
$v(t)$ is input into a band-pass filter. So, we should look at $V(f)$.
There are three terms in $v(t)$. Let's analyze them in the frequency domain separately.

(1) By the convolution-in-frequency property: $m^2(t) = m(t) \times m(t) \overset{\mathcal{F}}{\rightleftharpoons} M(f) * M(f)$.



In lecture, we have seen how to do self-convolution of a rectangular function. In this problem, $a = -54$, $b = 54$, $h = 1$, and the dummy variable is f instead of t. This gives



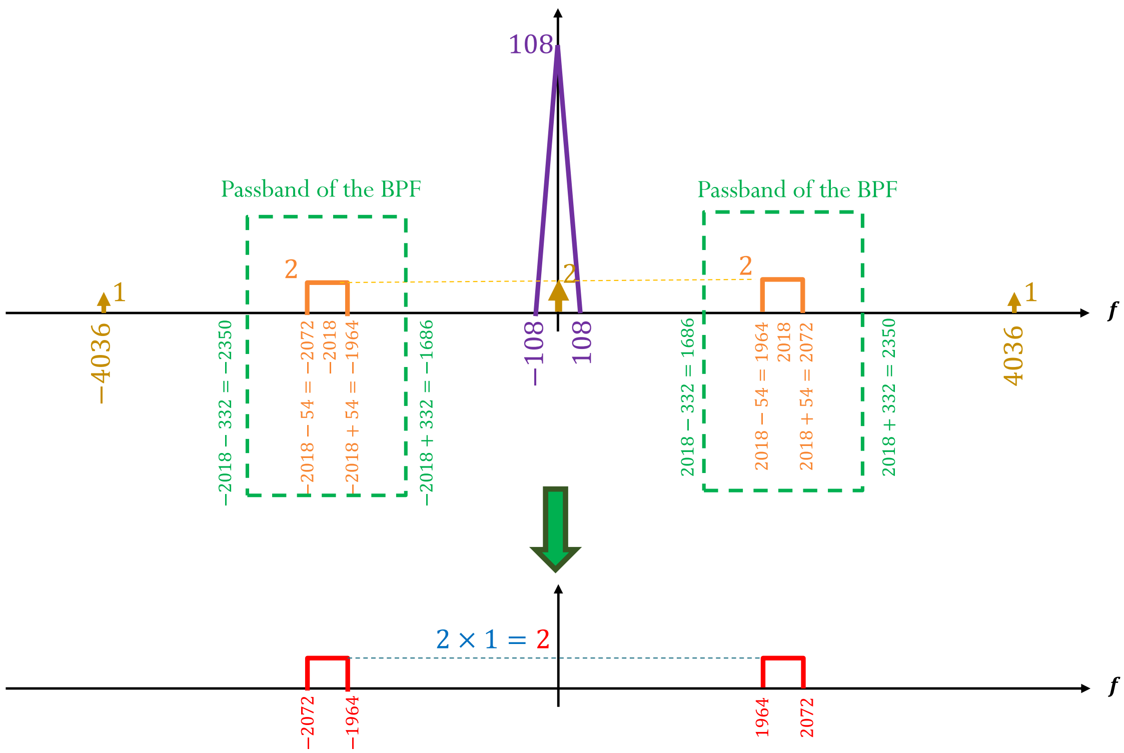
(Actually, the actual shape of the convolution is not as important as the fact that the result is band-limited to 128 Hz.

(2) From $\cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$, we have

$$4\cos^2(2\pi(2018)t) = 2 + 2\cos(2\pi(4036)t).$$

(3) Fourier transform of the term $4m(t)\cos(2\pi(2018)t)$ can be found by our modulation formula:

$$g(t)A\cos(2\pi(f_c)t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{A}{2}G(f-f_c) + \frac{A}{2}G(f-(-f_c)).$$



b. (Optional) Find $m(t)$ and $x(t)$.

$M(f)$ is a rectangular function in the frequency domain. So, $m(t)$ is a sinc function in the time domain. The area of the rectangular function is $1 \times 54 - (-54) = 108$. So,

$$m(t) = 108\,\mathrm{sinc}(2\pi f_m t),$$

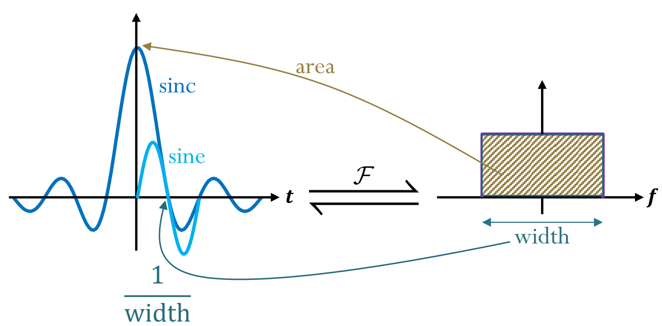
for some f_m.

The width of the rectangular function is $54 - (-54) = 108$. So, the first zero-crossing of the sinc function should occur at $t = \frac{1}{108}$. From $\mathrm{sinc}(\theta) = \frac{\sin(\theta)}{\theta}$, we see that the zero-crossing of $\mathrm{sinc}(\theta)$ are the same as the zero-crossings of $\sin(\theta)$ (except when $\theta = 0$).

Period of sine $= 2 \times \frac{1}{108}$.

Frequency of sine $= \frac{1}{\text{Period of sine}} = \frac{108}{2}$

So, $f_m = \frac{108}{2}$ and $m(t) = 108\,\mathrm{sinc}(108\pi t)$



Back in the analysis of $v(t)$, we saw that $4m(t)\cos(2\pi(2018)t)$ passes through the filter unchanged. So, $x(t) = 4m(t)\cos(2\pi(2018)t) = 432\,\mathrm{sinc}(108\pi t)\cos(4026\pi t)$.