

EES 351: Principles of Communications**2020/1**

HW 9 — Due: November 25, 11:59 PM

*Lecturer: Prapun Suksompong, Ph.D.***Instructions**

- (a) This assignment has 9 pages.
- (b) (1 pt) Two choices for submission:
- (i) Online submission via Google Classroom
 - PDF only. Paper size should be the same as the posted file.
 - Only for those who can directly work on the posted PDF file using devices with pen input.
 - No scanned work, photos, or screen capture.
 - Your file name should start with your 10-digit student ID: "5565242231 351 HW9.pdf"
 - (ii) Hardcopy submission: Work and write your answers **directly on a hardcopy of the posted file** (not on another blank sheet of paper).
- (c) (1 pt) Write your first name and the last three digits of your student ID in the spaces provided on the upper-right corner of this page.
- (d) (8 pt) Try to solve all problems.
- (e) Late submission will be heavily penalized.

Problem 1. Find the instantaneous frequency of the signal $g(t) = 3\sqrt{2} \cos(12t^3 + t^2)$ (a) at time $t = 0$ (b) at time $t = 2$

Problem 2. Recall that, in QAM system, the transmitted signal is of the form

$$x_{\text{QAM}}(t) = m_1(t) \sqrt{2} \cos(2\pi f_c t) + m_2(t) \sqrt{2} \sin(2\pi f_c t).$$

We want to express x_{QAM} in the form

$$x_{\text{QAM}}(t) = \sqrt{2}E(t) \cos(2\pi f_c t + \phi(t)),$$

where $E(t) \geq 0$ and $\phi(t) \in (-180^\circ, 180^\circ]$. (This shows that QAM can be expressed as a combination of amplitude modulation and phase modulation.)

Consider $m_1(t)$ and $m_2(t)$ plotted in Figure 9.1.

Draw the corresponding $E(t)$ and $\phi(t)$.

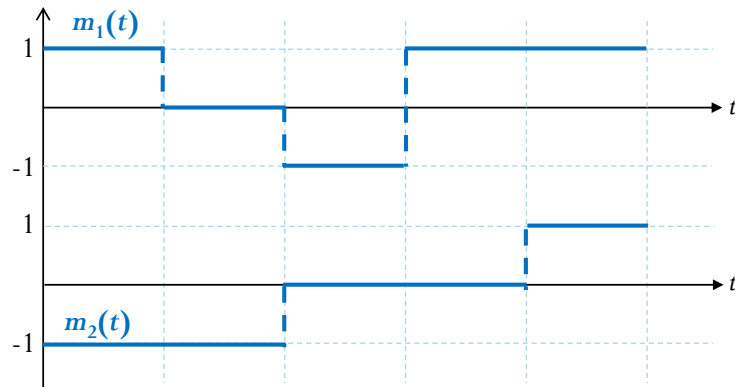


Figure 9.1: $m_1(t)$ and $m_2(t)$ for Problem 2

Problem 3. Consider the message $m(t)$ along with the carrier signal $\cos(2\pi f_c t + \phi)$ plotted in Figure 9.2. Note that $m(1.5) = 40$.

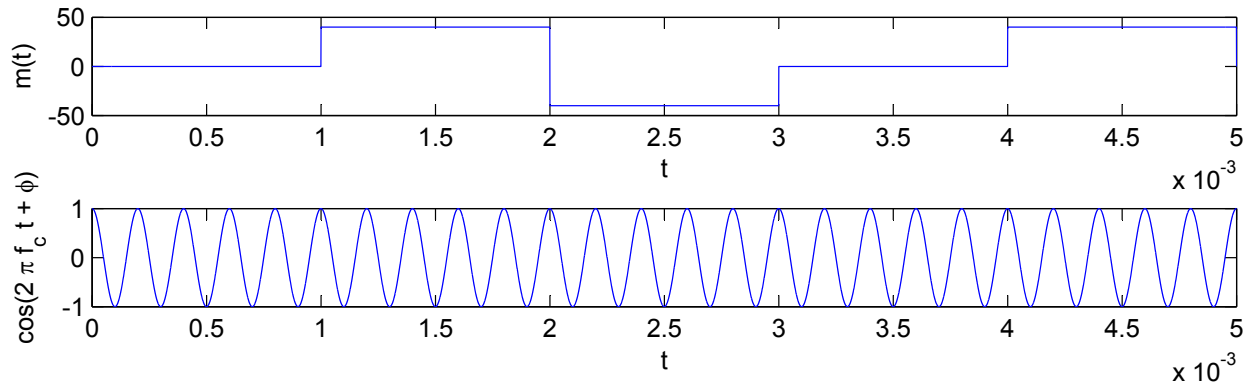


Figure 9.2: The message and the carrier signals for Problem 3.

- (a) Find the carrier frequency f_c from the plot. (Hint: It is an integer.)
- (b) Sketch the following signals. Make sure that (the unspecified parameter(s) are selected such that) the important “features” of the graphs can be seen clearly.
- (i) $x_{\text{AM}}(t) = (A + m(t)) \cos(2\pi f_c t + \phi)$ whose modulation index $\mu = 100\%$.

(ii) $x_{\text{FM}}(t) = A \cos\left(2\pi f_c t + \phi + 2\pi k_f \int_{-\infty}^t m(\tau) d\tau\right)$. Assume $m(t) = 0$ for $t < 0$.

(iii) $x_{\text{PM}}(t) = A \cos(2\pi f_c t + \phi + k_p m(t))$ with $k_p = \frac{\pi}{m_p}$.

Problem 4. Consider the FM transmitted signal

$$x_{\text{FM}}(t) = A \cos \left(2\pi f_c t + \phi + 2\pi k_f \int_{-\infty}^t m(\tau) d\tau \right),$$

where $f_c = 5$ [kHz], $A = 1$, and $k_f = 75$. The message $m(t)$ is shown in Figure 9.3.

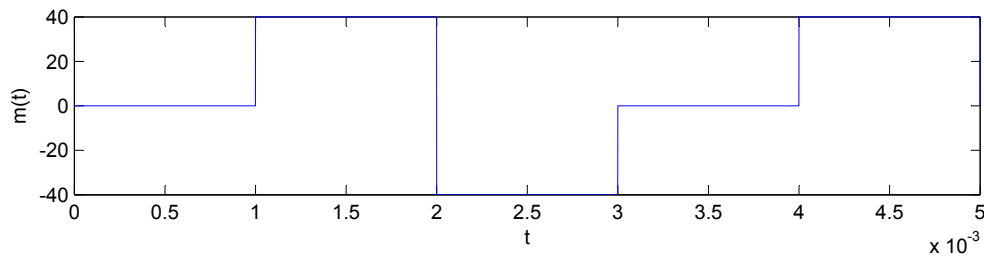


Figure 9.3: The message $m(t)$ for Problem 4

The magnitude spectrum $|X_{\text{FM}}(f)|$ is plotted in Figure 9.4.

(a) Find the values of f_1 , f_2 , and f_3 .

(b) Find the width W in Figure 9.4.

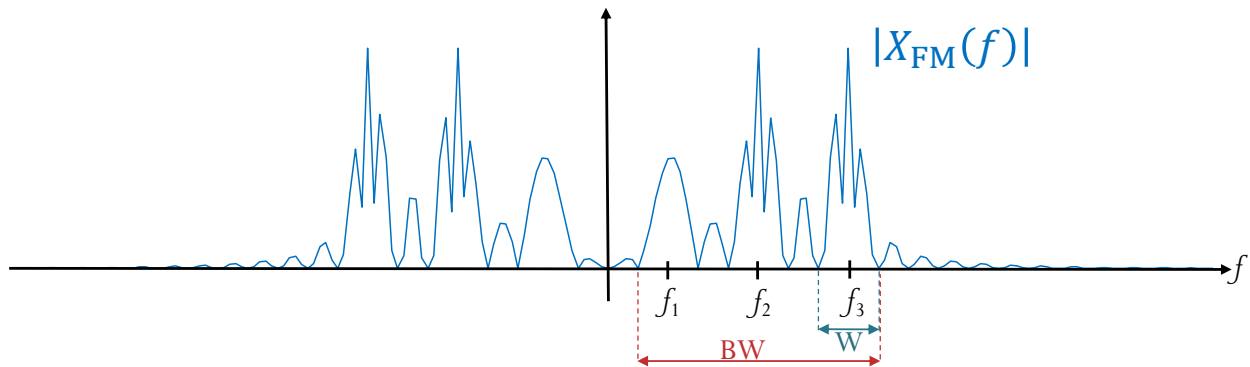


Figure 9.4: The magnitude spectrum $|X_{\text{FM}}(f)|$ for Problem 4

(c) Find the bandwidth denoted by BW in Figure 9.4.

Extra Questions

Here are some optional questions for those who want more practice.

Problem 5. Recall that, in QAM system, the transmitted signal is of the form

$$x_{\text{QAM}}(t) = m_1(t) \sqrt{2} \cos(2\pi f_c t) + m_2(t) \sqrt{2} \sin(2\pi f_c t).$$

In class, we have shown that

$$\text{LPF} \left\{ x_{\text{QAM}}(t) \sqrt{2} \cos(2\pi f_c t) \right\} = m_1(t).$$

Give a similar proof to show that

$$\text{LPF} \left\{ x_{\text{QAM}}(t) \sqrt{2} \sin(2\pi f_c t) \right\} = m_2(t).$$

Problem 6. In *quadrature amplitude modulation (QAM)* or *quadrature multiplexing*, two baseband signals $m_1(t)$ and $m_2(t)$ are transmitted simultaneously via the following QAM signal:

$$x_{\text{QAM}}(t) = m_1(t) \sqrt{2} \cos(\omega_c t) + m_2(t) \sqrt{2} \sin(\omega_c t).$$

An error in the phase or the frequency of the carrier at the demodulator in QAM will result in loss and interference between the two channels (cochannel interference).

In this problem, show that

$$\text{LPF} \left\{ x_{\text{QAM}}(t) \sqrt{2} \cos((\omega_c + \Delta\omega)t + \delta) \right\} = m_1(t) \cos((\Delta\omega)t + \delta) - m_2(t) \sin((\Delta\omega)t + \delta)$$

$$\text{LPF} \left\{ x_{\text{QAM}}(t) \sqrt{2} \sin((\omega_c + \Delta\omega)t + \delta) \right\} = m_1(t) \sin((\Delta\omega)t + \delta) + m_2(t) \cos((\Delta\omega)t + \delta).$$

Problem 7. In QAM system, the transmitted signal is of the form

$$x_{\text{QAM}}(t) = m_1(t) \sqrt{2} \cos(2\pi f_c t) + m_2(t) \sqrt{2} \sin(2\pi f_c t).$$

We want to express x_{QAM} in the form

$$x_{\text{QAM}}(t) = \sqrt{2}E(t) \cos(2\pi f_c t + \phi(t)),$$

where $E(t) \geq 0$ and $\phi(t) \in (-180^\circ, 180^\circ]$.

In each part below, we consider different examples of the messages $m_1(t)$ and $m_2(t)$.

- (a) Suppose $m_1(t) = \cos(2\pi Bt)$ and $m_2(t) = \sin(2\pi Bt)$ where $0 < B \ll f_c$. Find $E(t)$ and $\phi(t)$.
Hint: $\cos \alpha \cos \beta + \sin \alpha \sin \beta = \cos(\alpha - \beta)$

- (b) Suppose $m_1(t) = \cos(2\pi Bt)$ and $m_2(t) = 2 \sin(2\pi Bt)$. Let $f_c = 5$ and $B = 2$. Use MATLAB to plot the corresponding $E(t)$ and $\phi(t)$ from $t = 0$ to $t = 5$ [sec]. (Hint: the function `angle` or `atan2` will be helpful here.)

Problem 8. Consider a complex-valued signal $x(t)$ whose Fourier transform is $X(f)$.

(a) Find and simplify the Fourier transform of $x^*(t)$.

(b) Find and simplify the Fourier transform of $\text{Re}\{x(t)\}$.

- Hint: $x(t) + x^*(t) = ?$

Problem 9. Consider a (complex-valued) baseband signal $x_b(t) \xleftrightarrow[\mathcal{F}^{-1}]{\mathcal{F}} X_b(f)$ which is band-limited to B , i.e., $|X_b(f)| = 0$ for $|f| > B$. We also assume that $f_c \gg B$.

(a) The passband signal $x_p(t)$ is given by

$$x_p(t) = \sqrt{2} \text{Re} \left\{ e^{j2\pi f_c t} x_b(t) \right\}.$$

Find and simplify the Fourier transform of $x_p(t)$.

(b) Find and simplify

$$\text{LPF} \left\{ \sqrt{2} \left(\underbrace{\sqrt{2} \operatorname{Re} \left\{ e^{j2\pi f_c t} x_b(t) \right\}}_{x_p(t)} \right) e^{-j2\pi f_c t} \right\}.$$

Assume that the frequency response of the LPF is given by

$$H_{LP}(f) = \begin{cases} 1, & |f| \leq B \\ 0, & \text{otherwise.} \end{cases}$$