EES 351: Principles of Communications   **2020/1**

HW 8 — Due: November 11, 11:59 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 11 pages.

(b) (1 pt) Two choices for submission:

    (i) Online submission via Google Classroom
- PDF only. Paper size should be the same as the posted file.
- Only for those who can directly work on the posted PDF file using devices with pen input.
- No scanned work, photos, or screen capture.
- Your file name should start with your 10-digit student ID: "5565242231 351 HW4.pdf"

    (ii) Hardcopy submission: Work and write your answers **directly on a hardcopy of the posted file** (not on another blank sheet of paper).

(c) (1 pt) Write your first name and the last three digits of your student ID in the spaces provided on the upper-right corner of this page.

(d) (8 pt) Try to solve all problems.

(e) Late submission will be heavily penalized.

Problem 1. Consider the impulse train $G(f)$ shown on the right in Figure 8.1. Plot $g(t)$.
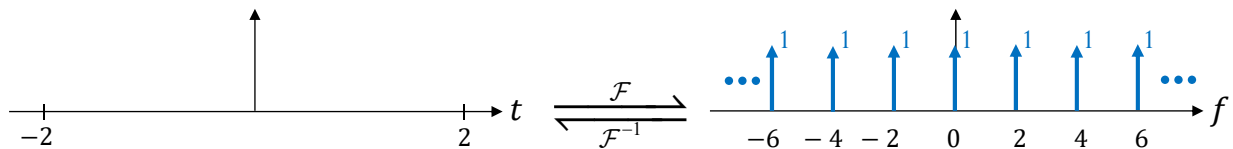

Figure 8.1: A train of impulses in the frequency domain

Problem 2. Consider a signal $r(t) = \sum_{k=-\infty}^{\infty} 2e^{j4\pi kt}$. Plot $r(t)$ and $R(f)$.
Hint: Don't try to actually plot each complex-expo. func. and add them. It is quite hopeless to determine their combination.

Problem 3. Consider an AM transmitter.

(a) Suppose the message is $m(t) = 4\cos(10\pi t)$ and the transmitted signal is

$$x_{\mathrm{AM}}(t) = A\cos(100\pi t) + m(t)\cos(100\pi t).$$

Find the value of A which yields the modulation index in each part below.

(i) $\mu = 50\%$
(ii) $\mu = 100\%$
(iii) $\mu = 150\%$

(b) Suppose the message is $m(t) = \alpha\cos(10\pi t)$ and the transmitted signal is

$$x_{\mathrm{AM}}(t) = 4\cos(100\pi t) + m(t)\cos(100\pi t).$$

Find the value of α which yields the modulation index in each part below.

(i) $\mu = 50\%$
(ii) $\mu = 100\%$
(iii) $\mu = 150\%$

Extra Questions

Here are some optional questions for those who want more practice.

Problem 4. You are asked to design a DSB-SC modulator to generate a modulated signal $km(t)\cos(2\pi f_c t)$, where $m(t)$ is a signal band-limited to B Hz. Figure 8.2 shows a DSB-SC modulator available in the stockroom. Note that, as usual, $\omega_c = 2\pi f_c$. The carrier generator available generates not $\cos(2\pi f_c t)$, but $\cos^3(2\pi f_c t)$. Explain whether you would be able to generate the desired signal using only this equipment. You may use any kind of filter you like. [Lathi and Ding, 2009, Q4.2-3]
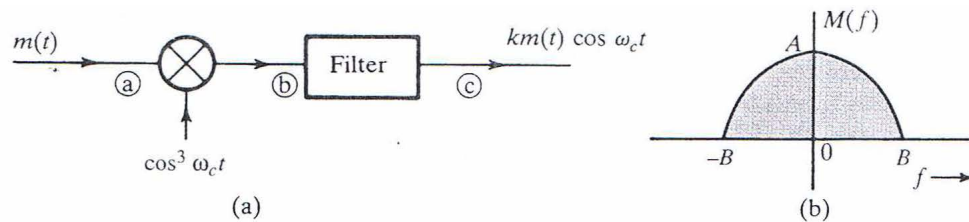

Figure 8.2: Problem 4

(a) A real-valued signal $r(t)$ that is even and periodic with period T_0 can be expanded using Fourier series into

$$r(t) = c_0 + a_1 \cos(2\pi f_0 t) + a_2 \cos(2\pi(2f_0)t) + a_3 \cos(2\pi(3f_0)t) + \cdots \qquad (8.1)$$

where $f_0 = \frac{1}{T_0}$. Consider the signal $r(t) = \cos^3(2\pi f_c t)$.

(i) Is it periodic?

(ii) Is it even?

(iii) Expand $r(t) = \cos^3(2\pi f_c t)$ into a linear combination of $\cos(2\pi(nf_c)t)$ as in (8.1) above.

(b) What kind of filter is required in Figure 8.2?

(c) Determine the signal spectra at points (b) and (c) in Figure 8.2, and indicate the frequency bands occupied by these spectra.

(d) What is the minimum usable value of f_c?

(e) Would this scheme work if the carrier generator output were $\cos^2(2\pi f_c t)$? Explain.

Problem 5. Consider the basic DSB-SC transceiver with time-delay channel presented in class. Recall that the input of the receiver is

$$x(t - \tau) = m(t - \tau)\sqrt{2}\cos(\omega_c(t - \tau))$$

where $m(t) \overset{\mathcal{F}}{\underset{\mathcal{F}^{-1}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. We also assume that $f_c \gg B$.

(a) Suppose that, at the receiver, we multiply by $\sqrt{2}\cos((\omega_c t) - \theta)$ instead of $\sqrt{2}\cos(\omega_c t)$ as illustrated in Figure 8.3. Assume

$$H_{LP}(f) = \begin{cases} 1, & |f| \leq B \\ 0, & \text{otherwise.} \end{cases}$$
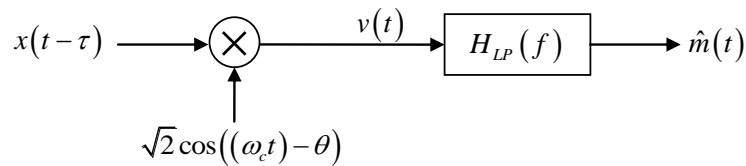
Find $\hat{m}(t)$ (the output of the LPF).

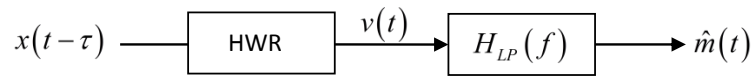

Figure 8.3: Receiver for Problem 5a

$$x(t-\tau) \longrightarrow \boxed{\text{HWR}} \xrightarrow{v(t)} \boxed{H_{LP}(f)} \longrightarrow \hat{m}(t)$$

Figure 8.4: Receiver for Problem 5b

(b) Use the same assumptions as part (a). However, at the receiver, instead of multiplying by $\sqrt{2}\cos((\omega_c t) - \theta)$, we pass $x(t - \tau)$ through a half-wave rectifier (HWR) as shown in Figure 8.4b.

Make an extra assumption that $m(t) \geq 0$ for all time t and that the half-wave rectifier input-output relation is described by a function $f(\cdot)$:

$$f(x) = \begin{cases} x, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

Find $\hat{m}(t)$ (the output of the LPF).

Problem 6 (M2011Q7). Suppose $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to W, i.e., $|M(f)| = 0$ for $|f| > W$. Consider a DSB-SC transceiver shown in Figure 8.5.
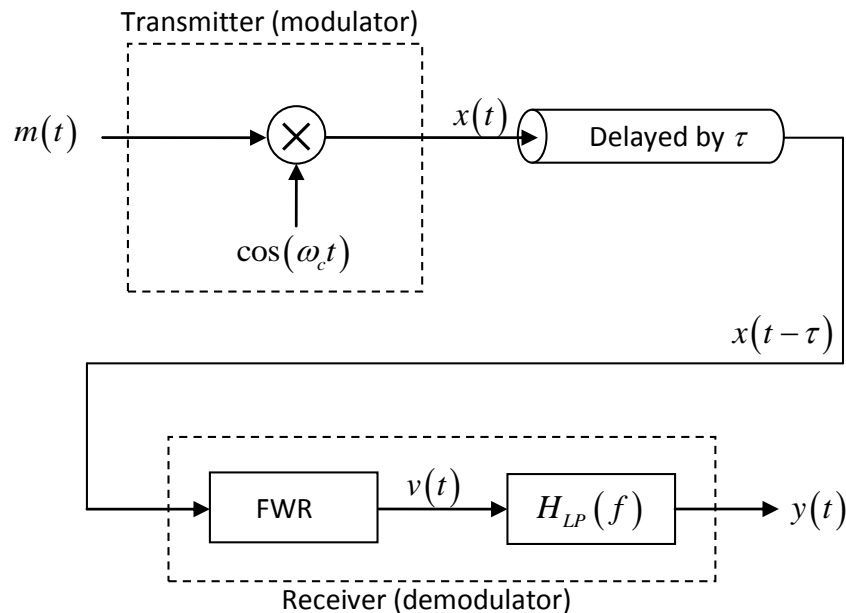

Figure 8.5: A DSB-SC transceiver

Also assume that $f_c \gg W$ and that $H_{LP}(f) = \begin{cases} 1, & |f| \leq W \\ 0, & \text{otherwise.} \end{cases}$

Make an extra assumption that $m(t) \geq 0$ for all time t and that the full-wave rectifier (FWR) input-output relation is described by a function $f_{FWR}(\cdot)$:

$$f_{FWR}(x) = \begin{cases} x, & x \geq 0, \\ -x, & x < 0. \end{cases}$$

(a) Recall that the **half**-wave rectifier input-output relation is described by a function $f_{HWR}(\cdot)$: $f_{HWR}(x) = \begin{cases} x, & x \geq 0, \\ 0, & x < 0. \end{cases}$ We have seen in Problem 5b that when the

receiver uses **half**-wave rectifier,

$$v\left(t\right) = x\left(t - \tau\right) \times g_{HWR}\left(t - \tau\right)$$

where $g_{HWR}\left(t\right) = 1\left[\cos\left(\omega_c t\right) \geq 0\right]$.

(i) The receiver in this question uses **full**-wave rectifier. Its $v(t)$ can be described in a similar manner; that is

$$v\left(t\right) = x\left(t - \tau\right) \times g_{FWR}\left(t - \tau\right).$$

Find $g_{FWR}\left(t\right)$. Hint: $g_{FWR}\left(t\right) = c_1 \times g_{HWR}\left(t\right) + c_2$ for some constants c_1 and c_2. Find these constants.

(ii) Recall that the Fourier series expansion of $g_{HWR}(t)$ is given by

$$g_{HWR}(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \frac{1}{7}\cos 7\omega_c t + \ldots\right).$$

Find the Fourier series expansion of $g_{FWR}(t)$.

(b) Find $y(t)$ (the output of the LPF).

Problem 7. Would the scheme in Problem 4 work if the carrier generator output were $\cos^n \omega_c t$ for any integer $n \geq 2$?

Problem 8 (M2011Q5)**.** In this question, you are provided with a partial proof of an important result in the study of Fourier transform. Your task is to figure out the quantities/-expressions inside the boxes labeled a,b,c, and d.

We start with a function $g(t)$. Then, we define $x(t) = \sum\limits_{\ell=-\infty}^{\infty} g(t - \ell T)$. It is a sum that involves $g(t)$. What you will see next is our attempt to find another expression for $x(t)$ in terms of a sum that involves $G(f)$.

To do this, we first write $x(t)$ as $x(t) = g(t) * \sum\limits_{\ell=-\infty}^{\infty} \delta(t - \ell T)$. Then, by the convolution-in-time property, we know that $X(f)$ is given by

$$X(f) = G(f) \times \boxed{ a } \sum_{\ell=-\infty}^{\infty} \delta\left(f + \boxed{ b }\right)$$

We can get $x(t)$ back from $X(f)$ by the inverse Fourier transform formula: $x(t) = \int\limits_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$. After plugging in the expression for $X(f)$ from above, we get

$$x(t) = \int\limits_{-\infty}^{\infty} e^{j2\pi f t} G(f) \boxed{ a } \sum_{\ell=-\infty}^{\infty} \delta\left(f + \boxed{ b }\right) df$$

$$= \boxed{ a } \int\limits_{-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} e^{j2\pi f t} G(f) \delta\left(f + \boxed{ b }\right) df.$$

By interchanging the order of summation and integration, we have

$$x(t) = \boxed{ a } \sum_{\ell=-\infty}^{\infty} \int\limits_{-\infty}^{\infty} e^{j2\pi f t} G(f) \delta\left(f + \boxed{ b }\right) df.$$

We can now evaluate the integral via the sifting property of the delta function and get

$$x(t) = \boxed{ a } \sum_{\ell=-\infty}^{\infty} e^{\boxed{ c }} G\left(\boxed{ d }\right).$$