Name ________________________ ID3 _ _ _

EES 351: Principles of Communications **2020/1**

HW 6 — Due: Not Due

Lecturer: Prapun Suksompong, Ph.D.

Problem 1. Consider the DSB-SC modem with no channel impairment shown in Figure 6.1. Suppose that the message is band-limited to $B = 3$ kHz and that $f_c = 100$ kHz.

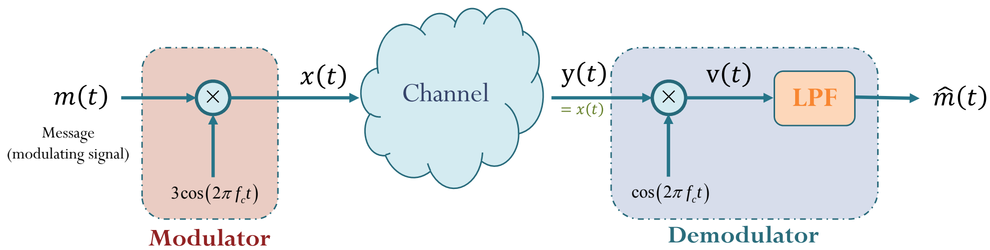

Figure 6.1: DSB-SC modem with no channel impairment

(a) Specify the frequency response $H_{LP}(f)$ of the LPF so that $\hat{m}(t) = m(t)$.

(b) Suppose the impluse response $h_{LP}(t)$ of the LPF is of the form $\alpha\,\mathrm{sinc}(\beta t)$. Find the constants α and β such that $\hat{m}(t) = m(t)$.

Problem 2. This question starts with a ***square-modulator*** for DSB-SC . Then, the use of the square-operation block is further explored on the receiver side of the system. [Doerschuk, 2008, Cornell ECE 320]

(a) Let $x(t) = A_c m(t)$ where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. Consider the block diagram shown in Figure 6.2.

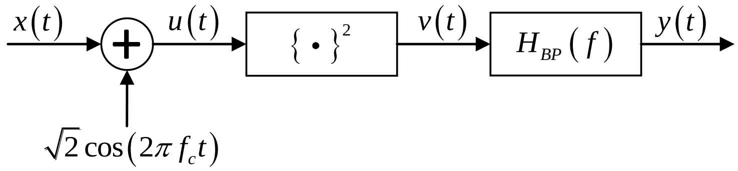

Figure 6.2: Block diagram for Problem 2a

Assume $f_c \gg B$ and

$$H_{BP}(f) = \begin{cases} 1, & |f - f_c| \leq B \\ 1, & |f + f_c| \leq B \\ 0, & \text{otherwise.} \end{cases}$$

The block labeled "$\{\cdot\}^2$" has output $v(t)$ that is the square of its input $u(t)$:

$$v(t) = u^2(t).$$

Find $y(t)$.

(b) The block diagram in part (a) provides a nice implementation of a modulator because it may be easier to build a squarer than to build a multiplier. Based on the successful use of a squaring operation in the modulator, we decide to use the same squaring operation in the demodulator. Let

$$x(t) = A_c m(t) \sqrt{2} \cos(2\pi f_c t)$$

where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. Again, assume $f_c \gg B$ Consider the block diagram shown in Figure 6.3.

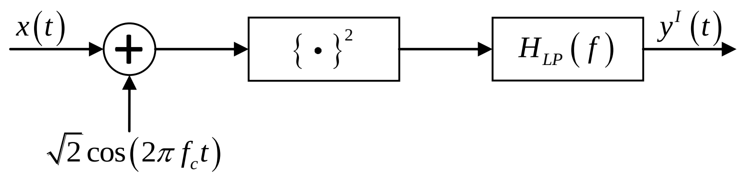

Figure 6.3: Block diagram for Problem 2b

Use

$$H_{LP}(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

Find $y^I(t)$. Does this block diagram work as a demodulator; that is, is $y^I(t)$ proportional to $m(t)$?

(c) Due to the failure in part (b), we have to think hard and it seems natural to consider also the block diagram with cos replaced by sin. Let

$$x(t) = A_c m(t) \sqrt{2} \cos(2\pi f_c t)$$

where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$ as in part (b). Again, assume $f_c \gg B$ Consider the block diagram shown in Figure 6.4.

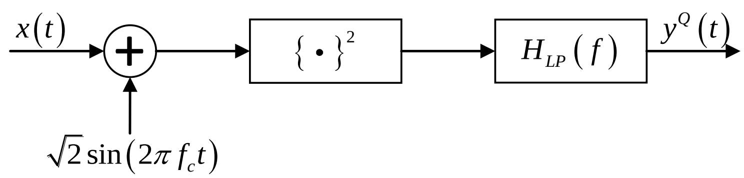

Figure 6.4: Block diagram for Problem 2c

As in part (b), use

$$H_{LP}(f) = \begin{cases} 1, & |f| \leq B \\ 0, & \text{otherwise.} \end{cases}$$

Find $y^Q(t)$.

(d) Use the results from parts (b) and (c). Draw a block diagram of a ***successful*** DSB-SC demodulator using squaring operations instead of multipliers.

Problem 3 (Cube modulator)**.** Consider the block diagram shown in Figure 6.5 where "$\{\cdot\}^3$" indicates a device whose output is the cube of its input.

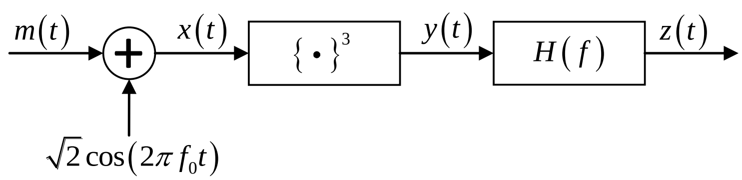

Figure 6.5: Block diagram for Problem 3. Note the use of f_0 instead of f_c.

Let $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ be bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$.

(a) Plot an $H(f)$ that gives $z(t) = m(t)\sqrt{2}\cos(2\pi f_c t)$. What is the gain in $H(f)$? What is the value of f_c? Notice that the frequency of the cosine is f_0 not f_c. You are supposed to determine f_c in terms of f_0.

(b) Let $M(f)$ be

$$M\left(f\right)=\begin{cases}1, & |f|\le B\\ 0, & \text{otherwise.}\end{cases}$$

(i) Plot $X(f)$.

(ii) Plot $Y(f)$. Hint:

$$M(f) * M(f) = \begin{cases} 2B - |f|, & |f| \leq 2B \\ 0, & \text{otherwise.} \end{cases}$$

Do not attempt to make an accurate plot or calculation for the Fourier transform of $m^3(t)$.

(iii) For your filter of part (a), plot $z(t)$.

[Doerschuk, 2008, Cornell ECE 320]