

EES 351: Principles of Communications

2020/1

HW 4 — Due: September 18, 11:59 PM

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Instructions

- (a) This assignment has 5 pages.
- (b) Some MATLAB scripts are available from http://www2.siit.tu.ac.th/prapun/ees351/EES351_2020_HW4_MATLAB.zip.
- (c) (1 pt) Two choices for submission:
- Online submission via Google Classroom
 - PDF only. Paper size should be the same as the posted file.
 - Only for those who can directly work on the posted PDF file using devices with pen input.
 - No scanned work, photos, or screen capture.
 - Your file name should start with your 10-digit student ID: "5565242231 351 HW4.pdf"
 - Hardcopy submission: Work and write your answers directly on a hardcopy of the posted file (not on another blank sheet of paper).
- (d) (1 pt) Write your first name and the last three digits of your student ID in the spaces provided on the upper-right corner of this page.
- (e) (8 pt) Try to solve all problems.
- (f) Late submission will be heavily penalized.

Problem 1 (M2011). The Fourier transform $X(f)$ for a signal $x(t)$ is shown in Figure 4.1.

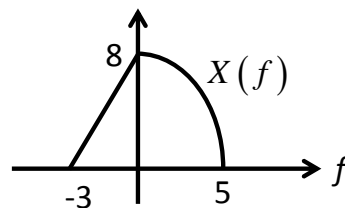


Figure 4.1: Plot of $X(f)$ for Problem 1.

Note that $x(t)$ is not provided directly. Only its Fourier transform $X(f)$ is plotted.

Let $g(t) = x(-2t)$ and $y(t) = x(4 - 2t)$. Carefully sketch $|G(f)|$ and $|Y(f)|$.

$$= x(-2(t-2)) = g(t-2)$$

$$= x(at) \text{ where } a = -2.$$

By the time-scaling property of Fourier transform,

$$G(f) = \frac{1}{|a|} X\left(\frac{f}{a}\right) = \frac{1}{2} X\left(-\frac{f}{2}\right)$$

$a = -2$

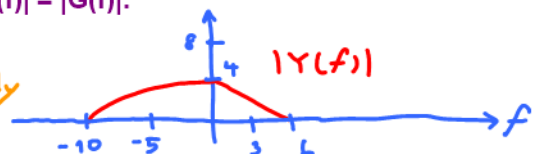
Hence,

$$|G(f)| = \left| \frac{1}{2} X\left(-\frac{f}{2}\right) \right| = \frac{1}{2} \left| X\left(-\frac{f}{2}\right) \right|$$

Annotations:
 - scaled vertically by a factor of $\frac{1}{2}$
 - no effect because $X(f)$ is already nonnegative.
 - expanded horizontally by a factor of 2
 - time reversal



Next, recall that time-shifting does not change the magnitude of the Fourier transform. Hence, $|Y(f)| = |G(f)|$.



Problem 2. ¹ Using MATLAB to find the (amplitude) spectrum² of a signal:

A signal $g(t)$ can often be expressed in analytical form as a function of time t , and the Fourier transform is defined as the integral of $g(t) \exp(-j2\pi ft)$. Often however, there is no analytical expression for a signal, that is, there is no (known) equation that represents the value of the signal over time. Instead, the signal is defined by measurements of some physical process. For instance, the signal might be the waveform at the input to the receiver, the output of a linear filter, or a sound waveform encoded as an mp3 file.

In all these cases, it is not possible to find the spectrum by analytically performing a Fourier transform. Rather, the discrete Fourier transform (or DFT, and its cousin, the more rapidly computable fast Fourier transform, or FFT) can be used to find the spectrum or frequency content of a measured signal. The MATLAB function `plotspect.m`, which plots the spectrum of a signal can be downloaded from our course website. Its help portion³ notes

```
% plotspect(x,t) plots the spectrum of the signal x
% whose values are sampled at time (in seconds) specified in t
```

- (a) The function `plotspect.m` should be straightforward to use. For instance, the spectrum of a rectangular pulse⁴ $g(t) = 1[0 \leq t \leq 2]$ can be found using:

```
% specrect.m plot the spectrum of a square wave
close all
Ts=1/100;           % time interval between adjacent samples
t=0:Ts:20;         % create a time vector
x=[t <= 2];       % rectangular pulse $1[0 \leq t \leq 2]$
plotspect(x,t)    % call plotspect to draw spectrum
xlim([-5,5])      % look only from f = -5 to f = 5 Hz
```

The output of `specrect.m` is shown in Figure 4.2. The top plot shows the first 20 seconds of $g(t)$. The bottom plot shows $|G(f)|$.

¹Based on [Johnson, Sethares, and Klein, 2011, Sec 3.1 and Q3.3].

²also referred to by “amplitude spectrum” or simply “spectrum”

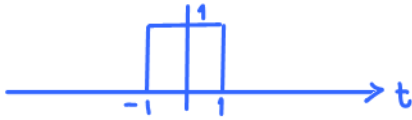
³You can view the “help” portion for a MATLAB function `xxx` by typing `help xxx` at the MATLAB prompt. If you get an error such as `xxx not found`, then this means either that the function does not exist, or that it needs to be moved into MATLAB’s search path.

⁴Here, we define a rectangular pulse using the indicator function $1[\cdot]$. This function outputs a 1 when the statement inside the square brackets is true; otherwise, it outputs a 0. For example,

$$1[0 \leq t \leq 2] = \begin{cases} 1, & 0 \leq t \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

Let

$$x(t) = g(t+1) = g(t-(-1))$$



Then, by the time-shift property,

$$X(f) = e^{j2\pi f(-(-1))} G(f)$$

and

$$|X(f)| = |G(f)|$$

$|e^{j\theta}| = 1$ for any θ

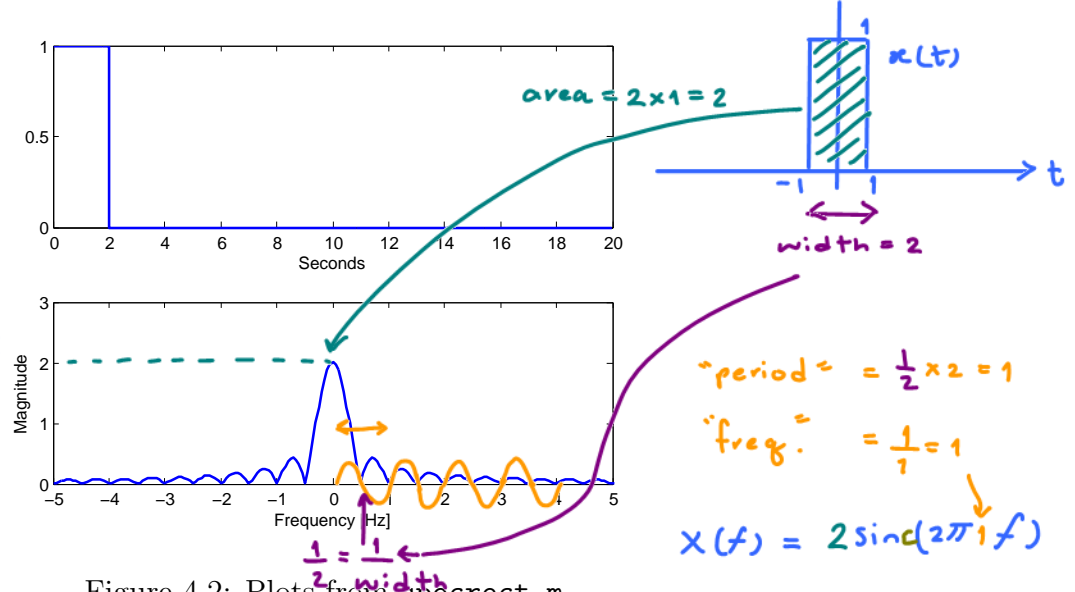


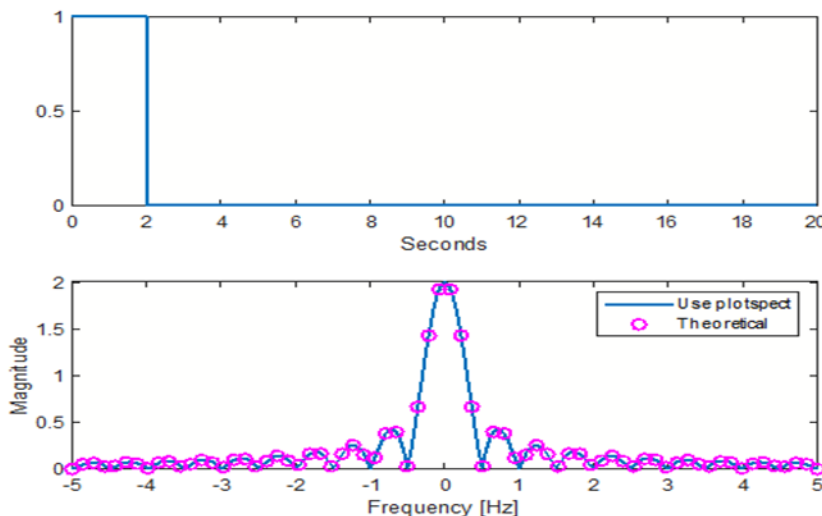
Figure 4.2: Plots from specrect.m

- (i) Use what we studied in class about the Fourier transform of a rectangular pulse (and the time-shift property) to find a simplified expression for $|G(f)|$.

$$|G(f)| = |X(f)| = 2|\text{sinc}(2\pi f)|$$

- (ii) Use MATLAB to plot your analytical expression derived in part (i). Put/paste the figure from MATLAB (not a hand-drawn one) here. Does your plot agree with the lower plot in Figure 4.2?

Caution: The built-in `sinc` function in MATLAB is defined using the normalized version. So, you will need to remove a factor of π from the argument of each sinc function found in part (i) when you type it into MATLAB.



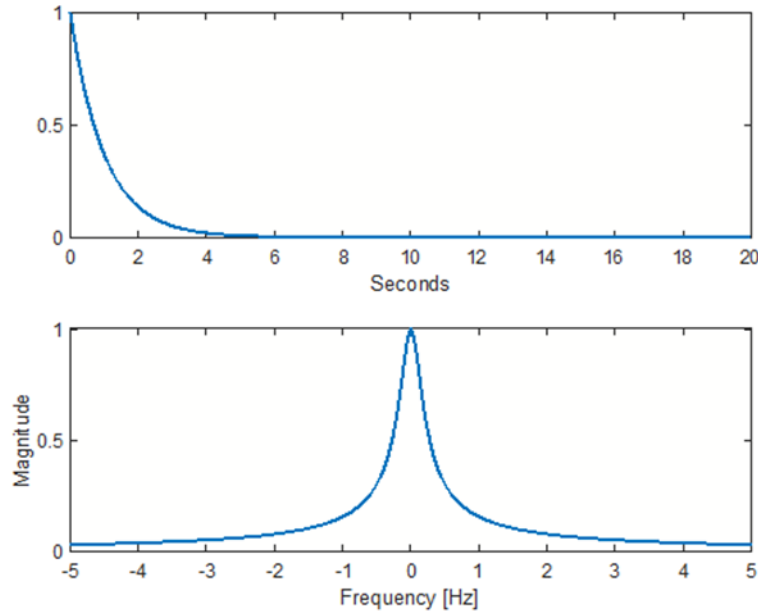
In the bottom part, the theoretical expression in part (i) is plotted using the "o" marks on top of the provided plot from specrect.m.

The two plots match perfectly.

(b) Now consider an exponential pulse

$$s(t) = e^{-t}u(t) \text{ where } u(t) \text{ is the unit step function.}$$

- (i) Modify the code in `spectrect.m` to show the (magnitude) spectrum $|S(f)|$. Put/paste the figure from MATLAB (not a hand-drawn one) here.



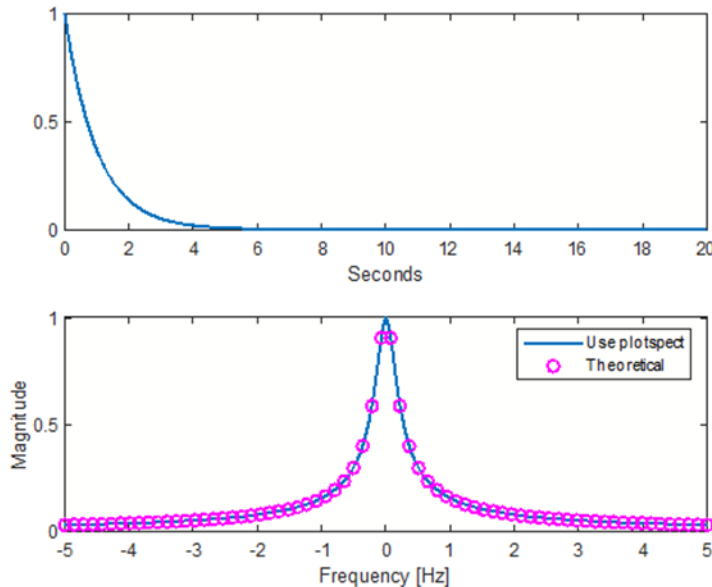
- (ii) Find $S(f)$ and $|S(f)|$ analytically. (Hopefully, you still remember how to integrate exponential function.)

$$\begin{aligned}
 S(f) &= \int_{-\infty}^{\infty} s(t)e^{-j2\pi ft} dt = \int_{-\infty}^{\infty} e^{-t}u(t)e^{-j2\pi ft} dt = \int_0^{\infty} e^{-(1+j2\pi f)t} dt \\
 &\quad \uparrow \\
 &\quad \text{direct FT formula} \\
 &= \frac{1}{-(1+j2\pi f)} e^{-(1+j2\pi f)t} \Big|_{t=0}^{\infty} = -\frac{1}{1+j2\pi f} (0-1) = \frac{1}{1+j2\pi f}
 \end{aligned}$$

Recall that the magnitude of a complex number $z = x + jy$ is $\sqrt{x^2 + y^2}$
 and that $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$

$$|S(f)| = \frac{1}{\sqrt{1+(2\pi f)^2}}$$

- (iii) Plot $|S(f)|$ from your analytical expression in part (ii) and compare with the plot in part (i). Put/paste the figure from MATLAB here.



The analytical expression in part (ii) is plotted using the “o” marks on top of the plot from spect.m from part (i). The two plots match perfectly.

- (iv) MATLAB can also perform symbolic manipulation when symbolic toolbox is installed. It can find the Fourier transform of a symbolic expression via the command `fourier`. Unfortunately, the `fourier` command use the ω -version of the definition. So, to convert the answer to the f -version, we also need to substitute $\omega = 2\pi f$. This is done automatically in our provided function `fourierf`. Run the file `SymbFourier.m`. Check whether you have the same result as part (ii).

Here is the result displayed on the command window:

```
>> SymbFourier
S =
1/(a + pi*f*2*i)
```

Setting the value of the variable “a” to 1, we have same result as in (ii).

Extra Question

Here is an optional question for those who want more practice.

Problem 3. Listen to the Fourier’s Song (Fouriers_Song.mp3) which can be downloaded from <http://sethares.engr.wisc.edu/mp3s/fourier.html>

Which properties of the Fourier Transform can you recognize from the song? List them here.