Name ________________________ ID3 _ _ _

EES 315: Probability and Random Processes **2020/1**

HW 6 — Due: October 22, 11:59 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 6 pages.

(b) (1 pt) Two choices for submission:

(i) Online submission via Google Classroom
- PDF only. Paper size should be the same as the posted file.
- Only for those who can directly work on the posted PDF file using devices with pen input.
- No scanned work, photos, or screen capture.
- Your file name should start with your 10-digit student ID: "5565242231 315 HW4.pdf"

(ii) Hardcopy submission: Work and write your answers **directly on a hardcopy of the posted file** (not on another blank sheet of paper).

(c) (1 pt) Write your first name and the last three digits of your student ID in the spaces provided on the upper-right corner of this page.

(d) (8 pt) Try to solve all problems. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

(e) Late submission will be heavily penalized.

Problem 1. Series Circuit: The circuit in Figure 6.1 operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates? [Montgomery and Runger, 2010, Ex. 2-32]

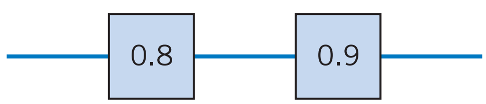

Figure 6.1: Circuit for Problem 1

Problem 2. In an experiment, A, B, C, and D are events with probabilities $P(A \cup B) = \frac{5}{8}$, $P(A) = \frac{3}{8}$, $P(C \cap D) = \frac{1}{3}$, and $P(C) = \frac{1}{2}$. Furthermore, A and B are disjoint, while C and D are independent.

(a) Find

  (i) $P(A \cap B)$

  (ii) $P(B)$

  (iii) $P(A \cap B^c)$

  (iv) $P(A \cup B^c)$

(b) Are A and B independent?

(c) Find

  (i) $P(D)$

  (ii) $P(C \cap D^c)$

  (iii) $P(C^c \cap D^c)$

  (iv) $P(C|D)$

(v) $P(C \cup D)$

(vi) $P(C \cup D^c)$

(d) Are C and D^c independent?

Problem 3. In this question, each experiment has equiprobable outcomes.

(a) Let $\Omega = \{1, 2, 3, 4\}$, $A_1 = \{1, 2\}$, $A_2 = \{1, 3\}$, $A_3 = \{2, 3\}$.

(i) Determine whether $P(A_i \cap A_j) = P(A_i)\, P(A_j)$ for all $i \neq j$.

(ii) Check whether $P(A_1 \cap A_2 \cap A_3) = P(A_1)\, P(A_2)\, P(A_3)$.

(iii) Are A_1, A_2, and A_3 independent?

(b) Let $\Omega = \{1, 2, 3, 4, 5, 6\}$, $A_1 = \{1, 2, 3, 4\}$, $A_2 = A_3 = \{4, 5, 6\}$.

(i) Check whether $P(A_1 \cap A_2 \cap A_3) = P(A_1)\, P(A_2)\, P(A_3)$.

(ii) Check whether $P(A_i \cap A_j) = P(A_i)\, P(A_j)$ for all $i \neq j$.

(iii) Are A_1, A_2, and A_3 independent?

Problem 4. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. Let A denote the event that the design color is red and let B denote the event that the font size is not the smallest one.

(a) Use classical probability to evaluate $P(A)$, $P(B)$ and $P(A \cap B)$. Show that the two events A and B are independent by checking whether $P(A \cap B) = P(A)P(B)$.

(b) Using the values of $P(A)$ and $P(B)$ from the previous part and the fact that $A \perp\!\!\!\perp B$, calculate the following probabilities.

(i) $P(A \cup B)$

(ii) $P(A \cup B^c)$

(iii) $P(A^c \cup B^c)$

[Montgomery and Runger, 2010, Q2-84]

Problem 5. The circuit in Figure 6.2 operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates? [Montgomery and Runger, 2010, Ex. 2-34]

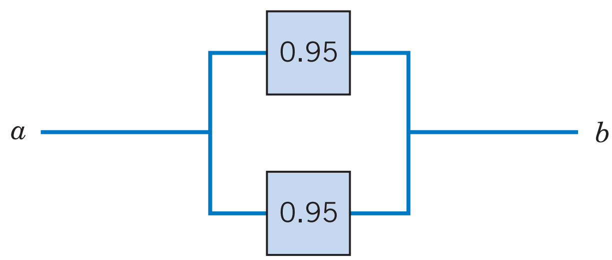

Figure 6.2: Circuit for Problem 5

Extra Questions

Here are some optional questions for those who want more practice.

Problem 6. Show that if A and B are independent events, then so are A and B^c, A^c and B, and A^c and B^c.

Problem 7. Anne and Betty go fishing. Find the conditional probability that Anne catches no fish given that at least one of them catches no fish. Assume they catch fish independently and that each has probability $0 < p < 1$ of catching no fish. [Gubner, 2006, Q2.62]

Hint: Let A be the event that Anne catches no fish and B be the event that Betty catches no fish. Observe that the question asks you to evaluate $P(A|(A \cup B))$.