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EES 315: Probability and Random Processes **2020/1**

HW 10 — Due: Not Due

Lecturer: Prapun Suksompong, Ph.D.

Problem 1 (Yates and Goodman, 2005, Q3.2.1)**.** The random variable X has probability density function

$$f_X(x) = \begin{cases} cx & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Use the pdf to find the following quantities.

(a) the unknown constant c

(b) $P[0 \le X \le 1]$

(c) $P[-1/2 \le X \le 1/2]$.

Problem 2 (Yates and Goodman, 2005, Q3.3.4)**.** The pdf of random variable Y is

$$f_Y(y) = \begin{cases} y/2 & 0 \le y < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $\mathbb{E}[Y]$.

(b) Find $\operatorname{Var} Y$.

Problem 3. The pdf of random variable V is

$$f_V(v) = \begin{cases} \frac{v+5}{72}, & -5 < v < 7, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is $\mathbb{E}[V]$?

(b) What is $\operatorname{Var}[V]$?

(c) What is $\mathbb{E}[V^3]$?

Problem 4 (Yates and Goodman, 2005, Q3.4.5)**.** X is a continuous uniform RV on the interval $(-5, 5)$.

(a) What is its pdf $f_X(x)$?

(b) What is $\mathbb{E}[X]$?

(c) What is $\mathbb{E}[X^5]$?

(d) What is $\mathbb{E}\left[e^X\right]$?

Problem 5. Suppose that the time to failure (in hours) of fans in a personal computer can be modeled by an exponential distribution with $\lambda = 0.0003$.

(a) What proportion of the fans will last at least 10,000 hours?

(b) What proportion of the fans will last at most 7000 hours?

[Montgomery and Runger, 2010, Q4-97]

Problem 6. Let a continuous random variable X denote the current measured in a thin copper wire in milliamperes. Assume that the probability density function of X is

$$f_X(x) = \begin{cases} 5, & 4.9 \le x \le 5.1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the probability that a current measurement is less than 5 milliamperes.

(b) Find the expected value of X.

(c) Find the variance and the standard deviation of X.

(d) Find the expected value of power when the resistance is 100 ohms?

Problem 7. Let X be a uniform random variable on the interval $[0, 1]$. Set

$$A = \left[0, \frac{1}{2}\right), \quad B = \left[0, \frac{1}{4}\right) \cup \left[\frac{1}{2}, \frac{3}{4}\right), \quad \text{and } C = \left[0, \frac{1}{8}\right) \cup \left[\frac{1}{4}, \frac{3}{8}\right) \cup \left[\frac{1}{2}, \frac{5}{8}\right) \cup \left[\frac{3}{4}, \frac{7}{8}\right).$$

Are the events $[X \in A], [X \in B]$, and $[X \in C]$ independent?

Problem 8 (Randomly Phased Sinusoid)**.** Suppose Θ is a uniform random variable on the interval $(0, 2\pi)$.

(a) Consider another random variable X defined by

$$X = 5\cos(7t + \Theta)$$

where t is some constant. Find $\mathbb{E}[X]$.

(b) Consider another random variable Y defined by

$$Y = 5\cos(7t_1 + \Theta) \times 5\cos(7t_2 + \Theta)$$

where t_1 and t_2 are some constants. Find $\mathbb{E}[Y]$.