EES 315: In-Class Exercise # 8

Date: 16 / 9 / 2020			
Name	ID (last 3 digits)		

Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

1. Consider a random experiment whose sample space is $\{a,b,c,d,e\}$ with outcome probabilities 0.1, 0.2, 0.2, 0.2, 0.3, respectively.

Here, it is given that $P(\{a\}) = 0.1$, $P(\{b\}) = 0.2$, $P(\{c\}) = 0.2$, $P(\{d\}) = 0.2$, $P(\{e\}) = 0.3$.

Let $A=\{a,b,c\}$, $B=\{b,c,d\}$, and $C=\{c,d,e\}$.

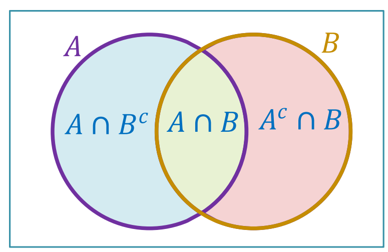
Find the following probabilities.

a. $P(A) = P(\{a,b,c\}) \underset{\text{finite additivity}}{=} P(\{a\}) + P(\{b\}) + P(\{c\}) = 0.1 + 0.2 + 0.2 = 0.5$.

b. $P\big((A\cup B)\cap C\big) = P(\{c,d\}) \underset{\text{finite additivity}}{=} P(\{c\}) + P(\{d\}) = 0.2 + 0.2 = 0.4$.

$(A \cup B) \cap C = \{a,b,c,d\} \cap \{c,d,e\} = \{c,d\}$

2. Suppose $P(A\cup B) = 0.9$, $P(A\cup B^c) = 0.8$, $P(A^c \cup B) = 0.6$.
Find $P(A\cap B)$.



Method 1:

$P(A \cap B^c) = 1 - P((A \cap B^c)^c) = 1 - P(A^c \cup B) = 1 - 0.6 = 0.4$.

$P(A^c \cap B) = 1 - P((A^c \cap B)^c) = 1 - P(A \cup B^c) = 1 - 0.8 = 0.2$.

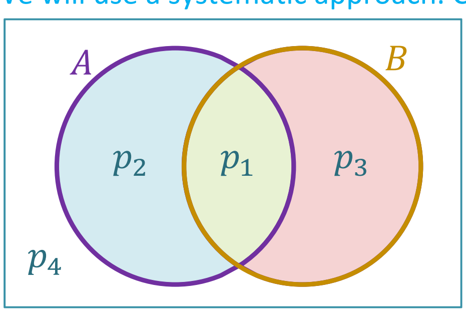
Observe that the three events $A \cap B^c$, $A \cap B$, and $A^c \cap B$ partition the event $A \cup B$.

By finite additivity, $P(A \cup B) = P(A \cap B^c) + P(A \cap B) + P(A^c \cap B)$.

Therefore, $P(A \cap B) = P(A \cup B) - P(A \cap B^c) - P(A^c \cap B) = 0.9 - 0.4 - 0.2 = 0.3$.

Method 2:

We will use a systematic approach. Consider the Venn diagram below.



We partition the sample space (Ω) into 4 parts.

Let p_i be the probability of the i^{th} part.

We are given that

$$P(A \cup B) = p_1 + p_2 + p_3 = 0.9,$$
$$P(A \cup B^c) = p_1 + p_2 + p_4 = 0.8, \text{ and}$$
$$P(A^c \cup B) = p_1 + p_3 + p_4 = 0.6.$$

Don't forget that we also have

$$p_1 + p_2 + p_3 + p_4 = 1.$$

Four equations to solve for four unknowns

Solving these gives $p_1 = 0.3,\ p_2 = 0.4,\ p_3 = 0.2,\ p_4 = 0.1$.

This gives $P(A \cap B) = p_1 = 0.3$.