EES 315: In-Class Exercise # 7

Date: 11 / 9 / 2020			
Name	ID (last 3 digits)		

Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

In this exercise, the answer in each problem should be reduced into just an integer.

1. In the expansion of $(x+y)^{2018}$, find the coefficient of $x^{2016}y^2$.

   From the **binomial theorem**, the coefficient of $x^r y^{n-r}$ in the expansion of $(x+y)^n$ is $\binom{n}{r}$.

   This is also the reason an alternative name for $\binom{n}{r}$ is **binomial coefficient.**

   Here, $n = 2018$ and $r = 2016$. Therefore,
$$\binom{n}{r} = \binom{2018}{2016} = \frac{2018!}{2016!\,2!} = \frac{2018 \times 2017}{2} = 2{,}035{,}153.$$

2. Find the number of solutions to $x_1 + x_2 + x_3 = 9$.

   Assume all variables are nonnegative integers.

   Recall that There are $\binom{r+n-1}{r} = \binom{r+n-1}{n-1}$ distinct n-tuples $(x_1, x_2, \dots, x_n)$ of nonnegative integers such that $x_1 + x_2 + \dots + x_n = r$.

   Here, $n = 3$ and $r = 9$. Therefore,
$$\binom{r+n-1}{n-1} = \binom{11}{2} = \frac{11 \times 10}{2} = 55.$$
   Alternatively, we try to distribute $r = 9$ (indistinguishable) copies of number one into $n = 3$ ordered room. To do this, we need $n-1 = 2$ walls. The permutation of $n-1$ walls and r ones is $\frac{n-1+r}{(n-1)!r!}$.

3. Suppose we sample 5 objects from a collection of 8 distinct objects. Calculate the number of different possibilities when the sampling is unordered and performed with replacement.

   Recall that there are $\binom{r+n-1}{r}$ possible unordered samples with replacement.

   Here, $n = 8$ and $r = 5$. Therefore,
$$\binom{r+n-1}{n-1} = \binom{12}{5} = 792.$$
   Alternatively, we can start from the counting problem. The important information about a sample is how many of each distinct object in our sample. There are $n = 8$ distinct objects. Let x_k be the number of the kth object in the sample. So, our counting problem is equivalent to finding the number of (nonnegative integer) solutions to the equation $x_1 + x_2 + \dots + x_n = r$.

   From our problem, the answer is $\binom{r+n-1}{r} = \binom{r+n-1}{n-1}$.