EES 315: In-Class Exercise # 6

Date: 9 / 9 / 2020			
Name	ID (last 3 digits)		

Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member.
     (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

1. Calculate the following quantities:

$$\binom{9}{2} = \frac{9!}{2!(9-2)!} = \frac{9!}{2!7!} = \frac{9\times8}{2\times1} = 36$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Here, $n = 9$ and $r = 2$.

$$(9)_2 = \underbrace{9 \times 8}_{\text{Product of 2 consecutive numbers. Start from 9. Reduce by 1 for the next number.}} = 72$$

Alternatively, we can use the formula:

$$(n)_r = \frac{n!}{(n-r)!}$$

Here, $n = 9$ and $r = 2$. So, $(9)_2 = \frac{9!}{7!}$

$$\binom{n}{n_1, n_2, \dots, n_r} = \frac{n!}{(n_1!)(n_2!)\cdots(n_r!)}$$

$$\binom{9}{2,2,5} = \frac{9!}{2!2!5!} = \frac{9\times8\times7\times6}{2!2!} = 756$$

2. Suppose we sample r 5 objects from a collection of n 9 distinct objects. Calculate the number of different possibilities when

   a. the sampling is ordered and performed with replacement

$$n^r = 9^5 = 59{,}049$$

   b. the sampling is ordered and performed without replacement

$$(n)_r = (9)_5 = \underbrace{9 \times 8 \times 7 \times 6 \times 5}_{\text{Product of 2 consecutive numbers. Start from 9. Reduce by 1 for the next number.}} = 15{,}120$$

   c. the sampling is unordered and performed without replacement

$$\binom{n}{r} = \binom{9}{5} = \frac{9!}{5!\,(9-5)!} = \frac{9!}{5!\,4!} = \frac{9 \times \overset{2}{\cancel{8}} \times 7 \times \cancel{6}}{\cancel{4} \times \cancel{3 \times 2} \times 1} = 126$$

3. Calculate the number of different results when we permute AAABBCCCC

$$n_1 = 3, n_2 = 2, n_3 = 4$$

$$n = n_1 + n_2 + n_3 = 9$$

$$\binom{n}{n_1, n_2, n_3} = \binom{9}{3,2,4} = \frac{9!}{3!\,2!\,4!} = \frac{9 \times \overset{4}{\cancel{8}} \times 7 \times \cancel{6} \times 5}{\cancel{(3 \times 2 \times 1)}(\cancel{4} \times 1)} = 1{,}260$$

Alternatively, we can use the same reasoning as in the lecture. Start by treating all characters as different. We have 9! ways of permuting them. Then, because the three A's are the same; permuting them should not create new samples. By the division principle, we divide 3! from our original count. Similarly, there are two B's and 4 C's. So, we divide 2! and 4! from our original count.

Don't forget to simplify your answers.