EES 315: In-Class Exercise # 5

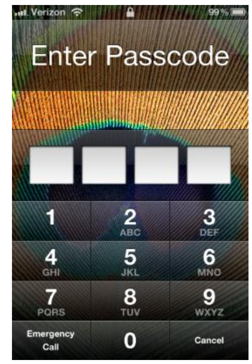
Date: 2 / 9 / 2020			
Name	ID (last 3 digits)		

Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

1) Consider 4-digit pin numbers.



a. How many different pin numbers are there?

To create a pin number, we need to specify four digits.
This can be done in four steps, one for each digit.
After the completion of one step, the next step has 10 choices (0 to 9).
By the **multiplication principle**, the total number of different 4-digit pin numbers is

$$10 \times 10 \times 10 \times 10 = 10^4 = 10{,}000.$$

b. How many different pin numbers are there that have at least one non-zero digit?

Let Ω be the set of all 4-digit pin numbers. From part (a), we know that $|\Omega| = 10^4$.
Let A be the set of all 4-digit pin numbers that have at least one non-zero digit.
The keyword "at least" suggests the use of the **subtraction principle**.
Note that a 4-digit pin number may have 0, 1, 2, 3, or 4 non-zero digits. So, the desired "at-least-one-non-zero-digit" condition covers many possibilities.
Therefore, it is easier to consider A^c which is the set of all 4-digit pin numbers that have no non-zero digit. Note that this means all four digits must be 0.
There is only one pin number that satisfies this condition: 0000.
Therefore, $|A^c| = 1$.
By the subtraction principle, because $A \subset \Omega$,

$$|A| = |\Omega| - |\Omega \backslash A| = |\Omega| - |A^c| = 10000 - 1 = 9{,}999.$$

Subtraction Principle: If $A \subset S$, then
$$|A| = |S| - |S \backslash A|.$$
When $S = \Omega$, we have $S \backslash A = \Omega \backslash A = A^c$ and
$$|A| = |\Omega| - |A^c|.$$

c. How many different pin numbers are there that have at least one zero?

Continue from part (b).
Let B be the set of all 4-digit pin numbers that have at least one zero.
Again, the keyword "at least" suggests the use of the **subtraction principle**.
Note that the number of 0 in a 4-digit pin number could be 0, 1, 2, 3, or 4.
So, the desired "at-least-one-zero" condition covers many possibilities.
Therefore, it is easier to consider B^c which is the set of all 4-digit pin numbers that use no 0; so, each digit can be any number from 1 to 9. By the **multiplication principle**,

$$|B^c| = 9 \times 9 \times 9 \times 9 = 9^4 = 6{,}561.$$

By the subtraction principle, because $B \subset \Omega$,

$$|B| = |\Omega| - |\Omega \backslash B| = |\Omega| - |B^c| = 10{,}000 - 6{,}561 = 3{,}439.$$