EES 315: In-Class Exercise # 3

Date: 2 6 / 0 8 / 2020			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **[ENRE] Explanation is not required for this exercise.**
3. Only one submission is needed for each group.
4. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
5. **Do not panic.**

[ENRE] Let

$A =$ the interval $[-\pi, \pi]$,

$B =$ the set of all real-valued x satisfying $\cos(x) = -x^2 - \pi$,

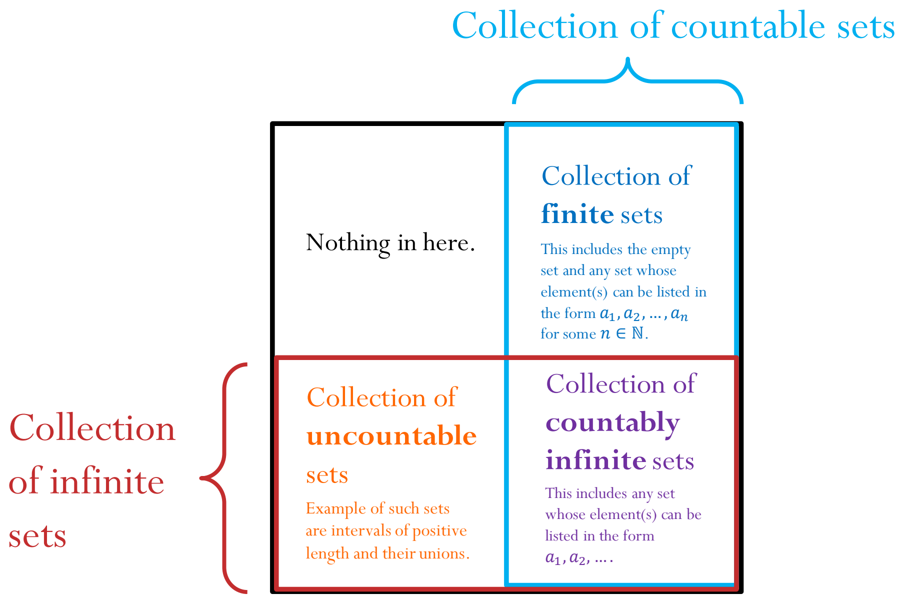
$C =$ the set of all real-valued x satisfying $\cos(x) < 0$, and

$D =$ the set of all positive integers that are divisible by 3.

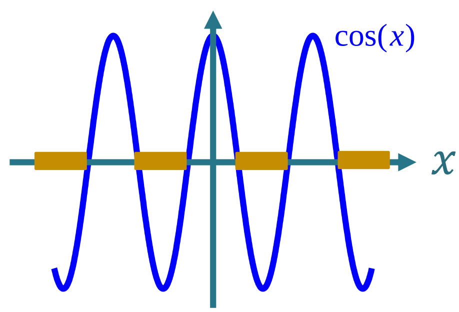
For each of the sets provided in the first column of the table below, indicate (by putting a Y(es) or an N(o) in each corresponding cell) whether it is "finite", "infinite", "countably infinite", "uncountable".

	Finite	Infinite	Countably Infinite	Uncountable
A	N	Y	N	Y
B	Y	N	N	N
C	N	Y	N	Y
D	N	Y	Y	N
$[-1,1] \cap [2,3]$	Y	N	N	N

First, we find the "key" type of each given set. (Figure 4 from the lecture notes is copied below.)



- Any interval with positive length is an uncountable set. Therefore, A is uncountable.
- We know that $x^2 \geq 0$. So, $-x^2 - \pi \leq -\pi$. Now, $-\pi < -1$. However, $\cos(x) \geq -1$. Therefore, the function "$-x^2 - \pi$" and the function "$\cos(x)$" will never intersect. Hence, $B = \varnothing$ which is finite.
- For set C, one can try to make a lousy plot of $\cos(x)$ and locate the x values that give $\cos(x) < 0$. This is shown below:



Observe that these x values correspond to a union of intervals all of which have positive length. Therefore, C is uncountable.

- $D = \{3, 6, 9, \ldots\}$ is countably infinite because its members can be listed in the form $a_1, a_2, a_3, \ldots$ by setting $a_k = 3k$.
- $[-1,1] \cap [2,3] = \varnothing$ which is finite.

	Finite	Infinite	Countably Infinite	Uncountable
A				Y
B	Y			
C				Y
D			Y	
$[-1,1] \cap [2,3]$	Y			

Then, we can apply the following reasoning:

- Any uncountable set is infinite. Any infinite set is not finite.
  Furthermore, any uncountable set is, by definition, not countable and therefore cannot be countably infinite.
  So, the answers for the corresponding row are N Y N Y.
- Any finite set cannot be infinite, countably infinite, nor uncountable.
  So, the answers for the corresponding row are Y N N N.
- Any countably infinite set is, by definition, infinite and hence not finite.
  Furthermore, any countably infinite set is, by definition, countable and hence not uncountable.
  So, the answers for the corresponding row are N Y Y N.