

# EES 315: In-Class Exercise # 20

## Instructions

1. Work alone or in a group of no more than three students. **The group cannot be the same as any of your former groups after the midterm.**
2. Only one submission is needed for each group.
3. You have two choices for submission:
  - (a) Online submission via Google Classroom
    - PDF only.
    - Only for those who can directly work on the posted files using devices with pen input.
    - Paper size should be the same as the posted file.
    - No scanned work, photos, or screen capture.
    - **Your file name should start with the 10-digit student ID of one member.**  
(You may add the IDs of other members, exercise #, or other information as well.)
  - (b) Hardcopy submission
4. **Do not panic.**

Date: 13 / 11 / 2020			
Name			ID <small>(last 3 digits)</small>

1. Continue from the previous exercise.

Find  $\mathbb{E}[X^2]$  and  $\text{Var}[X]$  for the random variable  $X$  defined in each part below:

$p_X(x)$	$\mathbb{E}[X]$	$\mathbb{E}[X^2]$	$\text{Var}[X]$								
$p_X(x) = \begin{cases} \frac{1}{6}x^2, & x \in \{-1, 1, 2\}, \\ 0, & \text{otherwise.} \end{cases}$ <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 2px;"><math>x</math></th> <th style="padding: 2px;"><math>p_X(x)</math></th> </tr> </thead> <tbody> <tr> <td style="padding: 2px;">-1</td> <td style="padding: 2px;"><math>\frac{1}{6}</math></td> </tr> <tr> <td style="padding: 2px;">1</td> <td style="padding: 2px;"><math>\frac{1}{6}</math></td> </tr> <tr> <td style="padding: 2px;">2</td> <td style="padding: 2px;"><math>\frac{2}{3}</math></td> </tr> </tbody> </table>	$x$	$p_X(x)$	-1	$\frac{1}{6}$	1	$\frac{1}{6}$	2	$\frac{2}{3}$	$\frac{4}{3}$	$= \sum_x x^2 p_X(x)$ $= \left((-1)^2 \times \frac{1}{6}\right) + \left(1^2 \times \frac{1}{6}\right) + \left(2^2 \times \frac{2}{3}\right)$ $= \frac{1}{6} + \frac{1}{6} + \frac{8}{3} = \frac{9}{3} = 3.$	$= \mathbb{E}[X^2] - (\mathbb{E}X)^2$ $= 3 - \left(\frac{4}{3}\right)^2 = \frac{11}{9}$ $\approx 1.2222$
$x$	$p_X(x)$										
-1	$\frac{1}{6}$										
1	$\frac{1}{6}$										
2	$\frac{2}{3}$										
$p_X(x) = \begin{cases} 0.4, & x = -1, 1, \\ 0.2, & x = 2, \\ 0, & \text{otherwise.} \end{cases}$ <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 2px;"><math>x</math></th> <th style="padding: 2px;"><math>p_X(x)</math></th> </tr> </thead> <tbody> <tr> <td style="padding: 2px;">-1</td> <td style="padding: 2px;">0.4</td> </tr> <tr> <td style="padding: 2px;">1</td> <td style="padding: 2px;">0.4</td> </tr> <tr> <td style="padding: 2px;">2</td> <td style="padding: 2px;">0.2</td> </tr> </tbody> </table>	$x$	$p_X(x)$	-1	0.4	1	0.4	2	0.2	0.4	$= \sum_x x^2 p_X(x)$ $= \left((-1)^2 \times 0.4\right) + \left(1^2 \times 0.4\right) + \left(2^2 \times 0.2\right)$ $= 0.4 + 0.4 + 0.8$ $= 1.6 = \frac{8}{5}$	$= \mathbb{E}[X^2] - (\mathbb{E}X)^2$ $= 1.6 - (0.4)^2$ $= 1.44 = \frac{36}{25}$
$x$	$p_X(x)$										
-1	0.4										
1	0.4										
2	0.2										