

EES 315: In-Class Exercise # 19

Instructions

1. Work alone or in a group of no more than three students. **The group cannot be the same as any of your former groups after the midterm.**
2. Only one submission is needed for each group.
3. You have two choices for submission:
 - (a) Online submission via Google Classroom
 - PDF only.
 - Only for those who can directly work on the posted files using devices with pen input.
 - Paper size should be the same as the posted file.
 - No scanned work, photos, or screen capture.
 - **Your file name should start with the 10-digit student ID of one member.**
(You may add the IDs of other members, exercise #, or other information as well.)
 - (b) Hardcopy submission
4. **Do not panic.**

Date: 11 / 11 / 2020			
Name			ID <small>(last 3 digits)</small>

1. Find the expected value of the random variable X defined in each part below:

(a)
$$p_X(x) = \begin{cases} cx^2, & x \in \{-1, 1, 2\}, \\ 0, & \text{otherwise.} \end{cases}$$

First, we need to solve for the value of the unknown constant c .
To be a pmf, we need " $\Sigma = 1$ ". So,

$$\begin{aligned} p_X(-1) + p_X(1) + p_X(2) &= 1 \\ c(-1)^2 + c(1)^2 + c(2)^2 &= 1 \\ c &= \frac{1}{6}. \end{aligned}$$

x	$p_X(x)$
-1	$c = \frac{1}{6}$
1	$c = \frac{1}{6}$
2	$4c = 4 \times \frac{1}{6} = \frac{2}{3}$

Therefore,

$$\begin{aligned} \mathbb{E}X &= \sum_x xp_X(x) \\ &= ((-1) \times p_X(-1)) + (1 \times p_X(1)) + (2 \times p_X(2)) \\ &= \left((-1) \times \frac{1}{6}\right) + \left(1 \times \frac{1}{6}\right) + \left(2 \times \frac{2}{3}\right) = \frac{4}{3} \approx 1.33. \end{aligned}$$

(b)
$$p_X(x) = \begin{cases} 0.4, & x = -1, 1, \\ c, & x = 2, \\ 0, & \text{otherwise.} \end{cases}$$

" $\Sigma = 1$ ":
$$\begin{aligned} p_X(-1) + p_X(1) + p_X(2) &= 1 \\ 0.4 + 0.4 + c &= 1 \\ c &= 0.2. \end{aligned}$$

x	$p_X(x)$
-1	0.4
1	0.4
2	$c = 0.2$

$$\mathbb{E}X = \sum_x xp_X(x) = ((-1) \times 0.4) + (1 \times 0.4) + (2 \times 0.2) = 0.4.$$

(c)
$$F_X(x) = \begin{cases} 0, & x < -1, \\ 0.3, & -1 \leq x < 2, \\ 1, & x \geq 2. \end{cases}$$

This cdf has two jumps; one is @ $x = -1$ and another one is @ $x = 2$.

The jump sizes are 0.4 and 0.6, respectively.

x	$p_X(x)$
-1	0.3
2	0.7

$$\mathbb{E}X = \sum_x xp_X(x) = ((-1) \times 0.3) + (2 \times 0.7) = 1.1.$$