# EES 315: In-Class Exercise # 14

## Instructions

1. Work alone or in a group of no more than three students. **The group cannot be the same as any of your former groups after the midterm.**
2. Only one submission is needed for each group.
3. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
4. **Do not panic.**

| Date: 21 / 10 / 2020 | | | |
|---|---|---|---|
| Name | ID (last 3 digits) | | |
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1. Consider a random experiment in which you roll a six-sided fair dice (whose faces are numbered 1-6). We define the following random variables from the outcomes of this experiment:

$$X(\omega) = \omega \quad \text{and} \quad Y(\omega) = (-1)^{\omega}.$$

   a. Find $P[X = 1]$.

   $X(\omega) = 1$ when $\omega = 1$. Therefore, $P[X = 1] = P(\{1\}) = \frac{1}{6}$.

   b. Find $P[Y = 1]$.

   $Y(\omega) = 1$ when $\omega = 2,4,6$.

   Therefore, $P[Y = 2] = P(\{2,4,6\}) = \frac{3}{6} = \frac{1}{2}$.

2. Consider a random experiment in which you roll a 10-sided fair dice (whose faces are numbered 0–9). Define a random variable $Z$ from the outcomes of this experiment by

$$Z(\omega) = (\omega - 5)^2.$$

$\Omega = \{0,1,2, \ldots, 9\}$

   a. Find $P[Z = 25]$.

   $Z(\omega) = 25$ when $(\omega - 5)^2 = 25$.

   $\omega = 5 \pm 5 = 0$ or 10. (10 is not in Ω.)

   Therefore, $P[Z = 25] = P(\{0\}) = \frac{1}{10}$.

   b. Find $P[Z > 5]$.

   Method 1:

   $Z(\omega) > 5$ when $(\omega - 5)^2 > 5$.

   $\omega > \underbrace{5 + \sqrt{5}}_{\approx 7.2361}$ or $\omega < \underbrace{5 - \sqrt{5}}_{\approx 2.7639}$

   $\omega = 8$ or $9$ $\quad$ $\omega = 0, 1,$ or $2$

   Therefore, $P[Z > 5] = P(\{0,1,2,8,9\}) = \frac{5}{10} = \frac{1}{2}$.

   Method 2: Because Ω is not large, it is possible to find $Z(\omega)$ for all $\omega$.

| $\omega$ | $\omega - 6$ | $(\omega - 6)^2$ | |
|---|---|---|---|
| 0 | -5 | 25 | |
| 1 | -4 | 16 | $> 5$ |
| 2 | -3 | 9 | |
| 3 | -2 | 4 | |
| 4 | -1 | 1 | |
| 5 | 0 | 0 | |
| 6 | 1 | 1 | |
| 7 | 2 | 4 | |
| 8 | 3 | 9 | $> 5$ |
| 9 | 4 | 16 | |