EES 315: In-Class Exercise # 13

Date: 14 / 10 / 2020			
Name	ID (last 3 digits)		

Instructions

1. Work alone or in a group of no more than three students. **The group cannot be the same as any of your former groups after the midterm.**
2. Only one submission is needed for each group.
3. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
4. **Do not panic.**

1) [Digital Communications] A certain binary-symmetric channel has a crossover probability (bit-error rate) of 0.4. Assume bit errors occur independently. Your answers for parts (a) and (b) should be of the form X.XXXX.

$p = 0.4$

a) Suppose we input bit sequence "1010101" into this channel.

The BSC is used $n = 7$ times (this is the number of bits in both the input and the output sequences.

1010101

i) What is the probability that the output is "1000001"?

This is similar to Example 6.54 in the lecture notes. However, note that p here is the probability of bit error. So, "success" here corresponds to the case that we have bit error; and "failure" corresponds to the case that the bit is unchanged when travels across the channel.

$$(1-p)\times(1-p)\times p\times(1-p)\times p\times(1-p)\times(1-p) = (1-p)^5\times p^2 = 0.6^5\times 0.4^2$$
$$= \frac{972}{78125} \approx 0.0124$$

$k = 4$

ii) What is the probability that exactly 4 bits are in error at the channel output?

See [6.55] and Example 6.60.a.i.

$$\binom{n}{k}p^k(1-p)^{n-k} = \binom{7}{4}0.4^4 0.6^3 = 35\times 0.4^4 0.6^3 = \frac{3024}{15625} \approx 0.1935$$

iii) What is the probability that there is at least one bit error at the channel output?

First, we consider the opposite case: the probability that there is no bit error is
$$\binom{n}{0}p^0(1-p)^{n-0} = \binom{7}{0}0.4^0 0.6^7 = 0.6^7.$$
Therefore, the probability that there is at least one bit error at the channel output is
$$1 - 0.6^7 \approx 0.9720.$$

b) Suppose we keep inputting bits into this channel. What is the probability that the **first** bit error at the output occurs on the fourth bit?

See Example 6.60.b.

Because the first bit error is on the fourth bit, this means the first three bits are unchanged across the channel and the fourth bit must be in error.
$$(1-p)\times(1-p)\times(1-p)\times p = (1-p)^3\times p = 0.6^3\times 0.4 = 0.0864$$