EES 315: In-Class Exercise # 12

Date: 9 / 10 / 2020			
Name	ID (last 3 digits)		

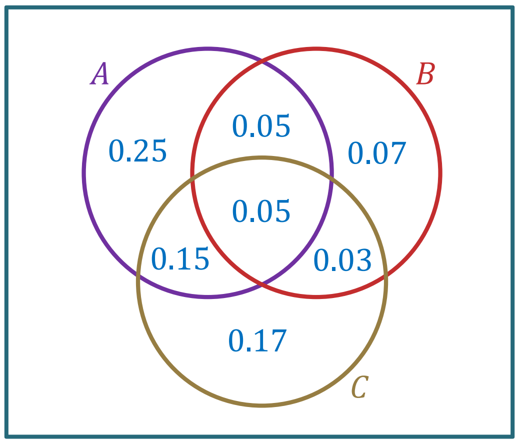
Instructions

1. Work alone or in a group of no more than three students. **The group cannot be the same as any of your former groups after the midterm.**
2. Only one submission is needed for each group.
3. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
4. **Do not panic.**

(1) Consider three events A, B, and C. Suppose

$$P(A^c \cap B \cap C) = 0.03, \quad P(A \cap B^c \cap C) = 0.15, \quad P(A \cap B \cap C^c) = 0.05,$$
$$P(A^c \cap B^c \cap C) = 0.17, \quad P(A^c \cap B \cap C^c) = 0.07, \quad P(A \cap B^c \cap C^c) = 0.25, \text{ and}$$
$$P(A \cap B \cap C) = 0.05$$



a. Are A, B, and C pairwise independent?

$P(A) = 0.25 + 0.15 + 0.05 + 0.05 = 0.5$
$P(B) = 0.05 + 0.05 + 0.07 + 0.03 = 0.2$
$P(C) = 0.05 + 0.15 + 0.03 + 0.17 = 0.4$

$P(A \cap B) = 0.05 + 0.05 = 0.1$
$P(A \cap C) = 0.15 + 0.05 = 0.2$
$P(B \cap C) = 0.05 + 0.03 = 0.08$

Checking pairwise independence for three events requires three conditions:

$P(A \cap B) = P(A)\,P(B)$ $\quad P(A \cap C) = P(A)P(C)$ $\quad P(B \cap C) = P(B)P(C)$

All three conditions are satisfied. Therefore, yes, the three events are pairwise independent.

b. Are A, B, and C independent?

Checking independence for three events requires four conditions. The first three conditions are the same as those for the pairwise independence which we have already checked in the previous part. Therefore, we only need to check the last condition:

$$P(A \cap B \cap C) \stackrel{?}{=} P(A)P(B)P(C).$$

Here, $P(A \cap B \cap C) = 0.05$. However, $P(A)P(B)P(C) = 0.5 \times 0.2 \times 0.4 = 0.04$.
Therefore, the last condition fails. So, no, the three events are not independent.