EES 315: In-Class Exercise # 11

Instructions

1. Work alone or in a group of no more than three students.
2. Only one submission is needed for each group.
3. You have two choices for submission:
   (a) Online submission via Google Classroom
   - PDF only.
   - Only for those who can directly work on the posted files using devices with pen input.
   - Paper size should be the same as the posted file.
   - No scanned work, photos, or screen capture.
   - Your file name should start with the 10-digit student ID of one member. (You may add the IDs of other members, exercise #, or other information as well.)

   (b) Hardcopy submission
4. **Do not panic.**

Date: 7 / 10 / 2020			
Name	ID (last 3 digits)		

(1) Suppose $P(A) = 0.5$ and $P(B) = 0.2$.

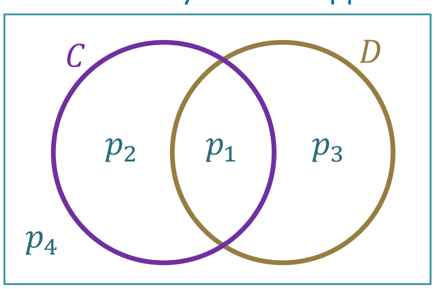
Find $P(A \cap B)$ to make events A and B independent.

By definition, events A and B are independent if and only if $P(A \cap B) = P(A)P(B)$.
Therefore, $P(A \cap B) = 0.5 \times 0.2 = 0.1$.

(2) Suppose $P(D) = 0.6$ and $P(C \cup D) = 0.8$.

Find $P(C \cap D)$ to make events C and D independent.

We will use a systematic approach. Consider the Venn diagram below.



As usual, for two events we partition the sample space (Ω) into 4 parts.

Let p_i be the probability of the i^{th} part.

Observe that $P(C \cap D)$ is p_1.

From the provided information, we know that

$$p_1 + p_3 = P(D) = 0.6. \quad (1)$$
$$p_1 + p_2 + p_3 = P(C \cup D) = 0.8. \quad (2)$$

As usual, we also know that

$$p_1 + p_2 + p_3 + p_4 = 1. \quad (3)$$

The requirement that events C and D must be independent means

$$P(C \cap D) = P(C)P(D)$$

This is equivalent to

$$p_1 = (p_1 + p_2)(p_1 + p_3). \quad (4)$$

Note that we now have four equations to solve for four unknowns. This should be possible to do.
Here, it requires only a few more steps to solve for p_1.

$$p_2 = 0.8 - 0.6 = 0.2$$

$$p_1 = (p_1 + 0.2)0.6.$$
$$p_1 = 0.3.$$

There are usually other "easier" solutions. However, they are not as systematic as the solution above.

For example, one can start with

$$P(C \cup D) = P(C) + P(D) - P(C \cap D).$$

Forcing the events C and D to be independent means we must have $P(C \cap D) = P(C)P(D)$.

So,

$$P(C \cup D) = P(C) + P(D) - P(C)P(D).$$

Plugging in the provided values, we have

$$0.8 = P(C) + 0.6 - P(C)0.6.$$

This gives

$$P(C) = 0.5.$$

Therefore,

$$P(C \cap D) = P(C)P(D) = 0.5 \times 0.6 = 0.3.$$