

# EES 315: In-Class Exercise # 1 - Sol

## Instructions

1. Separate into groups of no more than three students each.
2. **Explanation is not required for this exercise.**
3. **Do not panic.**

Date: <u>19/08/2020</u>			
Name			ID (last 3 digits)
Prapun			5 5 5

1. (7 pt) List **all** of Dr.Prapun's office hours during this week.

Hint: Check Google Calendar on the course website.

Many students are uncertain or shy about utilizing a professor's office hours. Don't let that be you! Each professor should hold office hours, a designated time in which the professor is available in his or her office to speak individually with students.

Date	Time	
	From	To
Monday: August 17, 2020	10:00	10:20
Tuesday: August 18, 2020	10:00	11:00
Tuesday: August 18, 2020	14:30	14:50
Wednesday: August 19, 2020	10:00	10:20
Wednesday: August 19, 2020	16:00	17:00
Thursday: August 20, 2020	14:30	14:50
Friday: August 21, 2020	13:40	14:00

2. (3 pt) (This one final task must be worked on individually, not as a group.)

Use your own Line account to send

your student id, followed by your full name, and then your nickname inside the parentheses

into the **EES315 Line group**.

Example: "50764555 Nadech Kugimiya (Barry)"

Remark: If you don't have a Line account, email the message as instructed above to [prapun@siit.tu.ac.th](mailto:prapun@siit.tu.ac.th).



# EES 315: In-Class Exercise # 2 - Sol

## Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **[ENRE] Explanation is not required for this exercise.**
3. Only one submission is needed for each group.
4. You have two choices for submission:
  - i. Online submission via Google Classroom
    - PDF only.
    - Only for those who can directly work on the posted files using devices with pen input.
    - Paper size should be the same as the posted file.
    - No scanned work, photos, or screen capture.
    - Your file name should start with the 10-digit student ID of one member.  
(You may add the IDs of other members, exercise #, or other information as well.)
  - ii. Hardcopy submission
5. **Do not panic.**

Date: <u>21/08/2020</u>			
Name			ID <small>(last 3 digits)</small>
Prapun			5 5 5

1. A fair coin is flipped six times. The results are:

H T T T H H.

Let  $A$  be the event that heads occurs.

Let  $R(A, n)$  denote the **relative frequency** of event  $A$  for the first  $n$  flips.

Calculate  $R(A, n)$  from  $n = 1$  to  $n = 6$ . **Write your answers in the form X.XX.**

$n$	1	2	3	4	5	6
$R(A, n)$	$\frac{1}{1} = 1.00$	$\frac{1}{2} = 0.50$	$\frac{1}{3} \approx 0.33$	$\frac{1}{4} = 0.25$	$\frac{2}{5} = 0.40$	$\frac{3}{6} \approx 0.50$

Note how the denominator is increased as we increase the value of  $n$ .

Recall that, to find the relative frequency of an event  $A$  for the first  $n$  trials, we use the formula:

$$R(A, n) = \frac{N(A, n)}{n} = \frac{\text{\#trials that } A \text{ occurs}}{n}$$

# EES 315: In-Class Exercise # 3

## Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **[ENRE] Explanation is not required for this exercise.**
3. Only one submission is needed for each group.
4. You have two choices for submission:
  - (a) Online submission via Google Classroom
    - PDF only.
    - Only for those who can directly work on the posted files using devices with pen input.
    - Paper size should be the same as the posted file.
    - No scanned work, photos, or screen capture.
    - Your file name should start with the 10-digit student ID of one member.  
(You may add the IDs of other members, exercise #, or other information as well.)
  - (b) Hardcopy submission
5. **Do not panic.**

Date: <u>26/08/2020</u>			
Name	ID <small>(last 3 digits)</small>		
Prapun	5	5	5

[ENRE] Let

$A =$  the interval  $[-\pi, \pi]$ ,

$B =$  the set of all real-valued  $x$  satisfying  $\cos(x) = -x^2 - \pi$ ,

$C =$  the set of all real-valued  $x$  satisfying  $\cos(x) < 0$ , and

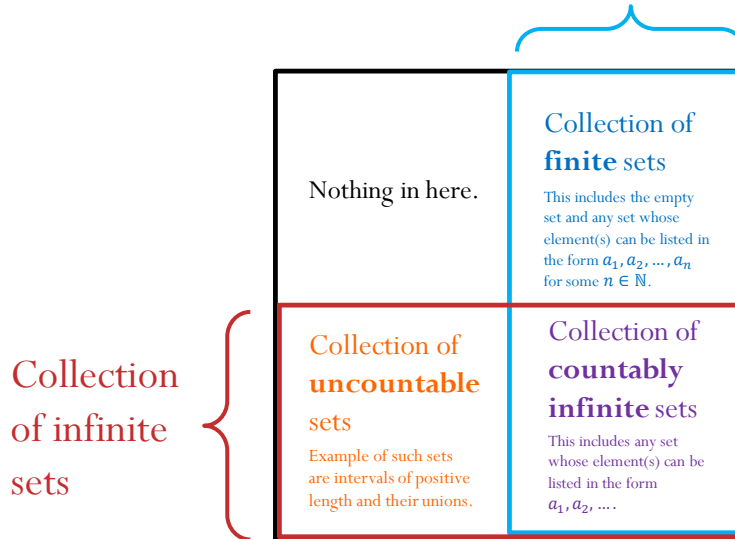
$D =$  the set of all positive integers that are divisible by 3.

For each of the sets provided in the first column of the table below, indicate (by putting a Y(es) or an N(o) in each corresponding cell) whether it is “finite”, “infinite”, “countably infinite”, “uncountable”.

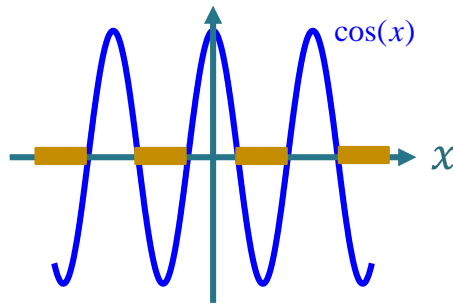
	Finite	Infinite	Countably Infinite	Uncountable
$A$	N	Y	N	Y
$B$	Y	N	N	N
$C$	N	Y	N	Y
$D$	N	Y	Y	N
$[-1,1] \cap [2,3]$	Y	N	N	N

First, we find the “key” type of each given set. (Figure 4 from the lecture notes is copied below.)

## Collection of countable sets



- Any interval with positive length is an uncountable set. Therefore,  $A$  is uncountable.
- We know that  $x^2 \geq 0$ . So,  $-x^2 - \pi \leq -\pi$ . Now,  $-\pi < -1$ . However,  $\cos(x) \geq -1$ . Therefore, the function “ $-x^2 - \pi$ ” and the function “ $\cos(x)$ ” will never intersect. Hence,  $B = \emptyset$  which is finite.
- For set  $C$ , one can try to make a lousy plot of  $\cos(x)$  and locate the  $x$  values that give  $\cos(x) < 0$ . This is shown below:



Observe that these  $x$  values correspond to a union of intervals all of which have positive length. Therefore,  $C$  is uncountable.

- $D = \{3, 6, 9, \dots\}$  is countably infinite because its members can be listed in the form  $a_1, a_2, a_3, \dots$  by setting  $a_k = 3k$ .
- $[-1, 1] \cap [2, 3] = \emptyset$  which is finite.

	Finite	Infinite	Countably Infinite	Uncountable
$A$				Y
$B$	Y			
$C$				Y
$D$			Y	
$[-1, 1] \cap [2, 3]$	Y			

Then, we can apply the following reasoning:

- Any uncountable set is infinite. Any infinite set is not finite.  
Furthermore, any uncountable set is, by definition, not countable and therefore cannot be countably infinite.  
So, the answers for the corresponding row are N Y N Y.
- Any finite set cannot be infinite, countably infinite, nor uncountable.  
So, the answers for the corresponding row are Y N N N.
- Any countably infinite set is, by definition, infinite and hence not finite.  
Furthermore, any countably infinite set is, by definition, countable and hence not uncountable.  
So, the answers for the corresponding row are N Y Y N.

# EES 315: In-Class Exercise # 4

## Instructions

- Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
- Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
- Only one submission is needed for each group.
- You have two choices for submission:
  - Online submission via Google Classroom
    - PDF only.
    - Only for those who can directly work on the posted files using devices with pen input.
    - Paper size should be the same as the posted file.
    - No scanned work, photos, or screen capture.
    - Your file name should start with the 10-digit student ID of one member.  
(You may add the IDs of other members, exercise #, or other information as well.)
  - Hardcopy submission
- Do not panic.**

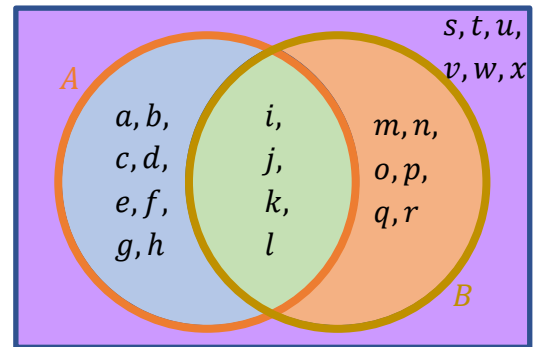
Date: 28 / 8 / 2020			
Name			ID (last 3 digits)

1) A random experiment has 24 **equiprobable** outcomes:

$$\{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x\}$$

Let  $A$  denote the event  $\{a, b, c, d, e, f, g, h, i, j, k, l\}$ , and

let  $B$  denote the event  $\{i, j, k, l, m, n, o, p, q, r\}$ .



Determine the following:

$$(a) P(B) = \frac{|B|}{|\Omega|} = \frac{10}{24} = \frac{5}{12} \approx 0.4167$$

$$|A \setminus B| = \{a, b, c, d, e, f, g, h\}$$

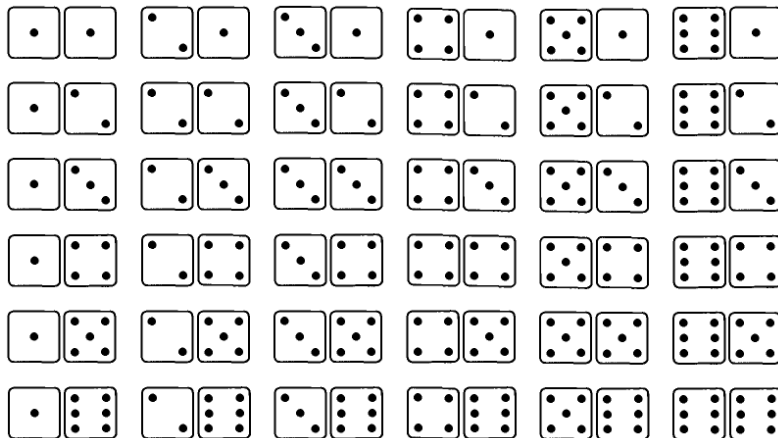
$$(b) P(A^c \cup B) = \frac{|A^c \cup B|}{|\Omega|} = \frac{|\Omega| - |A \setminus B|}{|\Omega|} = \frac{24 - 8}{24} = \frac{16}{24} = \frac{2}{3} \approx 0.67$$



$$\text{Alternatively, } A^c \cup B = \{i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x\}.$$

$$\text{Therefore, } P(A^c \cup B) = \frac{|A^c \cup B|}{|\Omega|} = \frac{16}{24}.$$

2) Roll two (fair) dice. What is the probability that the sum is less than 7?



Let  $A$  be the event that the sum is less than 7.

Consider all possible outcomes. The corresponding sums are

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

Among the 36 outcomes, only 15 of them satisfy the condition "sum < 7".

$$\text{Therefore, } P(A) = \frac{|A|}{|\Omega|} = \frac{15}{36} = \frac{5}{12} \approx 0.4167$$

# EES 315: In-Class Exercise # 5

## Instructions

- Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
- Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
- Only one submission is needed for each group.
- You have two choices for submission:
  - Online submission via Google Classroom
    - PDF only.
    - Only for those who can directly work on the posted files using devices with pen input.
    - Paper size should be the same as the posted file.
    - No scanned work, photos, or screen capture.
    - Your file name should start with the 10-digit student ID of one member.  
(You may add the IDs of other members, exercise #, or other information as well.)
  - Hardcopy submission
- Do not panic.**

Date: 2 / 9 / 2020			
Name			ID (last 3 digits)

### 1) Consider 4-digit pin numbers.

- a. How many different pin numbers are there?

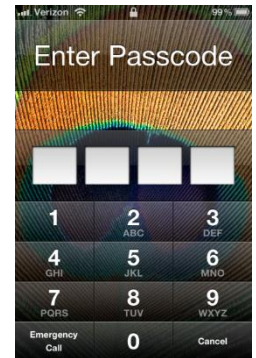
To create a pin number, we need to specify four digits.

This can be done in four steps, one for each digit.

After the completion of one step, the next step has 10 choices (0 to 9).

By the **multiplication principle**, the total number of different 4-digit pin numbers is

$$10 \times 10 \times 10 \times 10 = 10^4 = 10,000.$$



- b. How many different pin numbers are there that have at least one non-zero digit?

Let  $\Omega$  be the set of all 4-digit pin numbers. From part (a), we know that  $|\Omega| = 10^4$ .

Let  $A$  be the set of all 4-digit pin numbers that have at least one non-zero digit.

The keyword “at least” suggests the use of the **subtraction principle**.

Note that a 4-digit pin number may have 0, 1, 2, 3, or 4 non-zero digits. So, the desired “at-least-one-non-zero-digit” condition covers many possibilities.

Therefore, it is easier to consider  $A^c$  which is the set of all 4-digit pin numbers that have no non-zero digit. Note that this means all four digits must be 0.

There is only one pin number that satisfies this condition: 0000.

Therefore,  $|A^c| = 1$ .

By the subtraction principle, because  $A \subset \Omega$ ,

$$|A| = |\Omega| - |\Omega \setminus A| = |\Omega| - |A^c| = 10000 - 1 = 9,999.$$

**Subtraction Principle:** If  $A \subset S$ , then

$$|A| = |S| - |S \setminus A|.$$

When  $S = \Omega$ , we have  $S \setminus A = \Omega \setminus A = A^c$  and

$$|A| = |\Omega| - |A^c|.$$

- c. How many different pin numbers are there that have at least one zero?

Continue from part (b).

Let  $B$  be the set of all 4-digit pin numbers that have at least one zero.

Again, the keyword “at least” suggests the use of the **subtraction principle**.

Note that the number of 0 in a 4-digit pin number could be 0, 1, 2, 3, or 4.

So, the desired “at-least-one-zero” condition covers many possibilities.

Therefore, it is easier to consider  $B^c$  which is the set of all 4-digit pin numbers that use no 0; so, each digit can be any number from 1 to 9. By the **multiplication principle**,

$$|B^c| = 9 \times 9 \times 9 \times 9 = 9^4 = 6,561.$$

By the subtraction principle, because  $B \subset \Omega$ ,

$$|B| = |\Omega| - |\Omega \setminus B| = |\Omega| - |B^c| = 10,000 - 6,561 = 3,439.$$

# EES 315: In-Class Exercise # 6

## Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
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(You may add the IDs of other members, exercise #, or other information as well.)
  - (b) Hardcopy submission
5. **Do not panic.**

Date: 9 / 9 / 2020			
Name			ID <small>(last 3 digits)</small>

1. Calculate the following quantities:

$$\binom{9}{2} = \frac{9!}{2!(9-2)!} = \frac{9!}{2!7!} = \frac{9 \times 8}{2 \times 1} = 36$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Here,  $n = 9$  and  $r = 2$ .

$$(9)_2 = 9 \times 8 = 72$$

Product of 2 consecutive numbers.  
Start from 9. Reduce by 1 for the next number.

Alternatively, we can use the formula:

$$(n)_r = \frac{n!}{(n-r)!}$$

Here,  $n = 9$  and  $r = 2$ . So,  $(9)_2 = \frac{9!}{7!}$

$$\binom{n}{n_1, n_2, \dots, n_r} = \frac{n!}{(n_1!)(n_2!) \dots (n_r!)}$$

$$\binom{9}{2, 2, 5} = \frac{9!}{2!2!5!} = \frac{9 \times 8 \times 7 \times 6}{2!2!} = 756$$

2. Suppose we sample 5 objects from a collection of 9 distinct objects.

Calculate the number of different possibilities when

a. the sampling is ordered and performed with replacement

$$n^r = 9^5 = 59,049$$

b. the sampling is ordered and performed without replacement

$$(n)_r = (9)_5 = 9 \times 8 \times 7 \times 6 \times 5 = 15,120$$

Product of 2 consecutive numbers. Start from 9. Reduce by 1 for the next number.

c. the sampling is unordered and performed without replacement

$$\binom{n}{r} = \binom{9}{5} = \frac{9!}{5!(9-5)!} = \frac{9!}{5!4!} = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = 126$$

3. Calculate the number of different results when we permute AAABBCCCC

$$n_1 = 3, n_2 = 2, n_3 = 4$$

$$n = n_1 + n_2 + n_3 = 9$$

$$\binom{n}{n_1, n_2, n_3} = \binom{9}{3, 2, 4} = \frac{9!}{3!2!4!} = \frac{9 \times 8 \times 7 \times 6 \times 5}{(3 \times 2 \times 1)(2 \times 1)(4 \times 1)} = 1,260$$

Alternatively, we can use the same reasoning as in the lecture. Start by treating all characters as different. We have 9! ways of permuting them. Then, because the three A's are the same; permuting them should not create new samples. By the division principle, we divide 3! from our original count. Similarly, there are two B's and 4 C's. So, we divide 2! and 4! from our original count.

Don't forget to simplify your answers.



# EES 315: In-Class Exercise # 7

## Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Only one submission is needed for each group.
4. You have two choices for submission:
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(You may add the IDs of other members, exercise #, or other information as well.)
  - (b) Hardcopy submission
5. **Do not panic.**

Date: 11 / 9 / 2020			
Name			ID <small>(last 3 digits)</small>

In this exercise, the answer in each problem should be reduced into just an integer.

1. In the expansion of  $(x + y)^{2018}$ , find the coefficient of  $x^{2016}y^2$ .

From the **binomial theorem**, the coefficient of  $x^r y^{n-r}$  in the expansion of  $(x + y)^n$  is  $\binom{n}{r}$ .

This is also the reason an alternative name for  $\binom{n}{r}$  is **binomial coefficient**.

Here,  $n = 2018$  and  $r = 2016$ . Therefore,

$$\binom{n}{r} = \binom{2018}{2016} = \frac{2018!}{2016! 2!} = \frac{2018 \times 2017}{2} = 2,035,153.$$

2. Find the number of solutions to  $x_1 + x_2 + x_3 = 9$ .

Assume all variables are nonnegative integers.

Recall that There are  $\binom{r + n - 1}{r} = \binom{r + n - 1}{n - 1}$  distinct  $n$ -tuples  $(x_1, x_2, \dots, x_n)$  of nonnegative integers such that  $x_1 + x_2 + \dots + x_n = r$ .

Here,  $n = 3$  and  $r = 9$ . Therefore,

$$\binom{r + n - 1}{n - 1} = \binom{11}{2} = \frac{11 \times 10}{2} = 55.$$

Alternatively, we try to distribute  $r = 9$  (indistinguishable) copies of number one into  $n = 3$  ordered room.

To do this, we need  $n - 1 = 2$  walls. The permutation of  $n - 1$  walls and  $r$  ones is  $\frac{n-1+r}{(n-1)!r!}$ .

3. Suppose we sample 5 objects from a collection of 8 distinct objects. Calculate the number of different possibilities when the sampling is unordered and performed with replacement.

Recall that there are  $\binom{r + n - 1}{r}$  possible unordered samples with replacement.

Here,  $n = 8$  and  $r = 5$ . Therefore,

$$\binom{r + n - 1}{r} = \binom{12}{5} = 792.$$

Alternatively, we can start from the counting problem. The important information about a sample is how many of each distinct object in our sample. There are  $n = 8$  distinct objects. Let  $x_k$  be the number of the  $k$ th object in the sample. So, our counting problem is equivalent to finding the number of (nonnegative integer) solutions to the equation  $x_1 + x_2 + \dots + x_n = r$ .

From our problem, the answer is  $\binom{r + n - 1}{r} = \binom{r + n - 1}{n - 1}$ .

# EES 315: In-Class Exercise # 8

## Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Only one submission is needed for each group.
4. You have two choices for submission:
  - (a) Online submission via Google Classroom
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    - Only for those who can directly work on the posted files using devices with pen input.
    - Paper size should be the same as the posted file.
    - No scanned work, photos, or screen capture.
    - Your file name should start with the 10-digit student ID of one member.  
(You may add the IDs of other members, exercise #, or other information as well.)
  - (b) Hardcopy submission
5. **Do not panic.**

Date: 16 / 9 / 2020			
Name			ID (last 3 digits)

1. Consider a random experiment whose sample space is  $\{a, b, c, d, e\}$  with outcome probabilities 0.1, 0.2, 0.2, 0.2, 0.3, respectively.

Here, it is given that  $P(\{a\}) = 0.1,$   
 $P(\{b\}) = 0.2,$   
 $P(\{c\}) = 0.2,$   
 $P(\{d\}) = 0.2,$   
 $P(\{e\}) = 0.3.$

Let  $A = \{a, b, c\}$ ,  $B = \{b, c, d\}$ , and  $C = \{c, d, e\}$ .

Find the following probabilities.

a.  $P(A) = P(\{a, b, c\}) = P(\{a\}) + P(\{b\}) + P(\{c\}) = 0.1 + 0.2 + 0.2 = 0.5.$

↑  
finite additivity

b.  $P((A \cup B) \cap C) = P(\{c, d\}) = P(\{c\}) + P(\{d\}) = 0.2 + 0.2 = 0.4.$

↑ finite additivity

$(A \cup B) \cap C = \{a, b, c, d\} \cap \{c, d, e\} = \{c, d\}$

2. Suppose  $P(A \cup B) = 0.9$ ,  $P(A \cup B^c) = 0.8$ ,  $P(A^c \cup B) = 0.6$ .

Find  $P(A \cap B)$ .

Method 1:

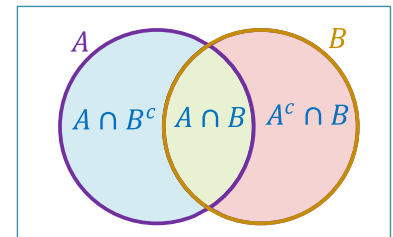
$P(A \cap B^c) = 1 - P((A \cap B^c)^c) = 1 - P(A^c \cup B) = 1 - 0.6 = 0.4.$

$P(A^c \cap B) = 1 - P((A^c \cap B)^c) = 1 - P(A \cup B^c) = 1 - 0.8 = 0.2.$

Observe that the three events  $A \cap B^c$ ,  $A \cap B$ , and  $A^c \cap B$  partition the event  $A \cup B$ .

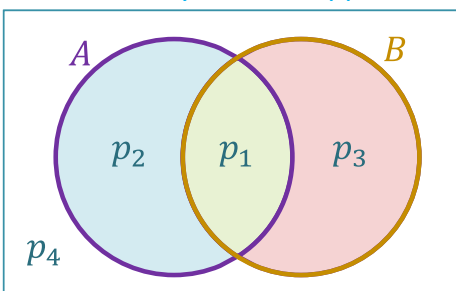
By finite additivity,  $P(A \cup B) = P(A \cap B^c) + P(A \cap B) + P(A^c \cap B)$ .

Therefore,  $P(A \cap B) = P(A \cup B) - P(A \cap B^c) - P(A^c \cap B) = 0.9 - 0.4 - 0.2 = 0.3.$



Method 2:

We will use a systematic approach. Consider the Venn diagram below.



We partition the sample space ( $\Omega$ ) into 4 parts.

Let  $p_i$  be the probability of the  $i^{\text{th}}$  part.

We are given that

$P(A \cup B) = p_1 + p_2 + p_3 = 0.9,$

$P(A \cup B^c) = p_1 + p_2 + p_4 = 0.8,$  and

$P(A^c \cup B) = p_1 + p_3 + p_4 = 0.6.$

Four equations to solve for four unknowns

Don't forget that we also have

$p_1 + p_2 + p_3 + p_4 = 1.$

Solving these gives  $p_1 = 0.3$ ,  $p_2 = 0.4$ ,  $p_3 = 0.2$ ,  $p_4 = 0.1.$

This gives  $P(A \cap B) = p_1 = 0.3.$

# EES 315: In-Class Exercise # 9

## Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **[ENRE] Explanation is not required for this exercise.**
3. Only one submission is needed for each group.
4. You have two choices for submission:
  - (a) Online submission via Google Classroom
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(You may add the IDs of other members, exercise #, or other information as well.)
  - (b) Hardcopy submission
5. **Do not panic.**

Date: 18 / 9 / 2020			
Name			ID <small>(last 3 digits)</small>

Here, it is given that

$$P(\{a\}) = 0.2,$$

$$P(\{b\}) = 0.3,$$

$$P(\{c\}) = 0.3,$$

$$P(\{d\}) = 0.2.$$

1. Consider a random experiment whose sample space is  $\{a, b, c, d\}$  with outcome probabilities 0.2, 0.3, 0.3, and 0.2, respectively. Let  $A = \{a, b, c\}$ , and  $B = \{c, d\}$ . Find the following probabilities.

$P(A B) = \frac{P(A \cap B)}{P(B)} = \frac{P(\{c\})}{P(\{c, d\})} = \frac{0.3}{0.3+0.2} = 0.6$	$A \cap B^c = A \setminus B = \{a, b\}$ $P(A B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{P(\{a, b\})}{1 - P(B)} = \frac{0.2+0.3}{1-0.5} = 1$
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2. Consider the following sequences of 1s and 0s which summarize the data obtained from 16 testees in a disease testing experiment.

D:	0	$\frac{1}{1}$	$\frac{1}{2}$	0	$\frac{0}{1}$	0	$\frac{0}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{0}{3}$	$\frac{1}{7}$	0	$\frac{1}{8}$	$\frac{1}{9}$
TP:	0	0	0	0	$\frac{1}{1}$	0	$\frac{1}{2}$	$\frac{1}{3}$	0	0	$\frac{1}{4}$	$\frac{1}{5}$	0	0	$\frac{1}{6}$	$\frac{1}{7}$

The results in the  $i$ -th column are for the  $i$ -th testee. The D row indicates whether each of the testees actually has the disease under investigation. The TP row indicates whether each of the testees is tested positive for the disease. Numbers “1” and “0” correspond to “True” and “False”, respectively.

Suppose we randomly pick a testee from this pool of 16 persons. Let  $D$  be the event that this selected person actually has the disease. Let  $T_p$  be the event that this selected person is tested positive for the disease.

Find the following probabilities. There are 15 testees; so the sample space is finite. We “randomly” pick one testee; so it makes sense to assume that each testee has equal chance of being selected. Therefore, classical probability can be applied here.

$P(D) = \frac{9}{16}$ <p style="text-align: center; color: blue;">Among the 16 testees, 9 have the disease.</p>	$P(T_p) = \frac{7}{16}$ <p style="text-align: center; color: green;">Among the 16 testees, 7 test positive.</p>
$P(T_p \cap D) = \frac{4}{16} = \frac{1}{4}$ <p style="text-align: center; color: purple;">Among the 16 testees, 4 have the disease and test positive.</p>	$P(T_p \cap D^c) = \frac{3}{16}$ <p style="text-align: center; color: orange;">Among the 16 testees, 3 test positive but do not have the disease.</p>

In each part below, additional information about the selected testee is available; this additional information is given in the condition part. With such information, find the corresponding conditional probability.

$P(T_p   D) = \frac{4}{9}$ <p style="text-align: center; color: purple;">Among the 9 testees who have the disease, 4 test positive.</p>	$P(T_p   D^c) = \frac{3}{7}$ <p style="text-align: center; color: orange;">Among the <math>16 - 9 = 7</math> testees who don't have the disease, 3 test positive.</p>
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Alternatively, 
$$P(T_p | D) = \frac{P(T_p \cap D)}{P(D)} = \frac{\frac{4}{16}}{\frac{9}{16}} = \frac{4}{9}$$

$$P(T_p | D^c) = \frac{P(T_p \cap D^c)}{P(D^c)} = \frac{\frac{3}{16}}{1 - \frac{9}{16}} = \frac{\frac{3}{16}}{\frac{7}{16}} = \frac{3}{7}$$

# EES 315: In-Class Exercise # 10

## Instructions

1. Work alone or in a group of no more than three students. For group work, **the group cannot be the same as any of your former groups in this class.**
2. **[ENRE] Explanation is not required for this exercise.**
3. Only one submission is needed for each group.
4. You have two choices for submission:
  - (a) Online submission via Google Classroom
    - PDF only.
    - Only for those who can directly work on the posted files using devices with pen input.
    - Paper size should be the same as the posted file.
    - No scanned work, photos, or screen capture.
    - Your file name should start with the 10-digit student ID of one member.  
(You may add the IDs of other members, exercise #, or other information as well.)
  - (b) Hardcopy submission

Date: 23 / 9 / 2020			
Name			ID <small>(last 3 digits)</small>

1. Consider a medical diagnostic test for detecting an illness (L). Suppose the probability that the test correctly identifies someone with the illness as positive (+) is 0.9, and the probability that the test correctly identifies someone without the illness as negative (-) is 0.8. The probability of finding the illness in the general population is 0.1.

(a) Find  $P(-|L)$ , the conditional probability that a randomly-chosen person tests negative given that the person does have the illness.

We are given three pieces of information:  $P(L) = 0.1$ ,  $P(+|L) = 0.9$ ,  $P(-|L^c) = 0.8$ .

Recall that  $P(A^c|B) = 1 - P(A|B)$ .

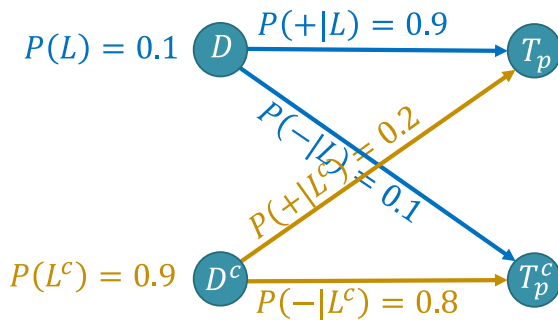
Therefore,  $P(-|L) = 1 - P(+|L) = 1 - 0.9 = 0.1$ .

- (b) A random person takes this test.  
What is the probability that this person tests positive?

$$P(+|L^c) = 1 - P(-|L^c) = 1 - 0.8 = 0.2$$

$$P(L^c) = 1 - 0.1 = 0.9$$

We can also get this expression directly from the total probability theorem.



$$\begin{aligned}
 P(+) &= P(+ \cap L) + P(+ \cap L^c) \\
 &= P(+|L)P(L) + P(+|L^c)P(L^c) \\
 &= 0.9 \times 0.1 + 0.2 \times 0.9 \\
 &= 0.27
 \end{aligned}$$

- (c) A random person takes this test, and the test result is positive.  
What is the probability that this person has the illness?

$$P(L|+) = \frac{P(L \cap +)}{P(+)} = \frac{P(+|L)P(L)}{P(+)} = \frac{0.9 \times 0.1}{0.27} = \frac{1}{3} \approx 0.33.$$

We can also get this expression directly by "Form 1" of the Bayes' theorem