ECS 455: In-Class Exercise # 9

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Date: 15/03/2017

Name	ID (last 3 digits)		
Prapun	5	5	5

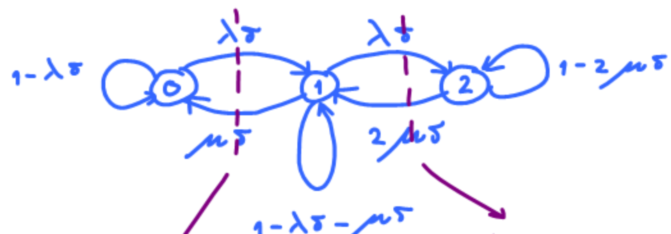
Consider a system with two channels. We would like to find the call blocking probability via the Markov chain method. For each of the following models, **draw the Markov chain** via discrete time approximation. Don't forget to label all the arrows. Then, use balance equations to find (1) the **steady-state probabilities** and then (2) the long-term **call blocking probability**. Don't forget to draw the corresponding boundaries and indicate which balance equations they represent.

1. ***Erlang B*** model: Assume that the *total* call request rate is 6 calls per hour and the average call duration is 30 mins.

$\lambda = 6 \frac{\text{calls}}{\text{hr}} = \frac{6}{60} \frac{\text{calls}}{\text{min}} = \frac{1}{10} \frac{\text{calls}}{\text{min}}$

$\frac{1}{\mu} = 30 \text{ min} = \frac{1}{2} \text{hr}.$

$A = \frac{\lambda}{\mu} = \lambda \times \frac{1}{\mu} = 6 \times \frac{1}{2} = 3$ Erlangs



$p_0 \lambda\delta = p_1 \mu\delta$

$p_1 = A p_0 = 3p_0$

$p_1 \lambda\delta = 2\mu\delta p_2$

$p_2 = \frac{A}{2} p_1 = \frac{A^2}{2} p_0$

$= \frac{3}{2} p_1 = \frac{9}{2} p_0$

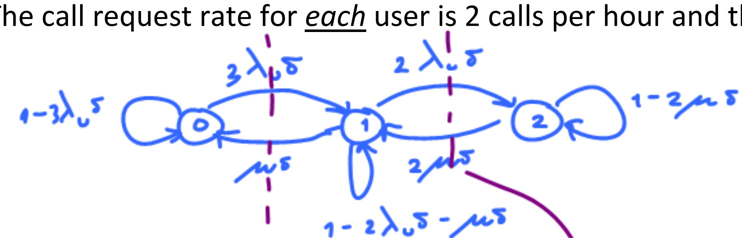
$p_0 + p_1 + p_2 = 0 \Rightarrow p_0 = \frac{1}{1 + A + \frac{A^2}{2}} = \frac{1}{1+3+\frac{3^2}{2}} = \frac{2}{17} \approx 0.118 \Rightarrow p_1 = 3 \times \frac{2}{17} = \frac{6}{17} \approx 0.353, \quad P_b = p_2 = \frac{3}{2} \times \frac{6}{17} = \frac{9}{17} \approx 0.529.$

2. ***Engset*** model: Assume that there are 3 users. The call request rate for *each* user is 2 calls per hour and the average call duration is 30 mins.

$\lambda_u = 2 \frac{\text{calls}}{\text{hr}}$ ← observe that this is $\frac{\lambda}{3}$

$\frac{1}{\mu} = 30 \text{ min} = \frac{1}{2} \text{hr}.$ ← same as before.

$A_u = \frac{\lambda_u}{\mu} = 2 \times \frac{1}{2} = 1$ Erlang.



$p_0 3\lambda_u\delta = p_1 \mu\delta$

$p_1 = 3A_u p_0$

$p_1 2\lambda_u\delta = p_2 2\mu\delta$

$p_2 = A_u p_1 = 3A_u^2 p_0$

$p_0 + p_1 + p_2 = 1 \Rightarrow p_0 = \frac{1}{1 + 3A_u + 3A_u^2} = \frac{1}{7} \approx 0.143$

$p_1 = 3A_u p_0 = 3p_0 = \frac{3}{7} \approx 0.429$

$p_2 = 3A_u^2 p_0 = 3p_0 = \frac{3}{7} \approx 0.429$

$P_b = \frac{\frac{3}{7} \times \lambda_u\delta}{\frac{1}{7} \times 3\lambda_u\delta + \frac{3}{7} \times 2\lambda_u\delta + \frac{3}{7} \times \lambda_u\delta} = \frac{3}{3+6+3}$

$= \frac{1}{4} \approx 0.25$