

ECS 455: In-Class Exercise # 8

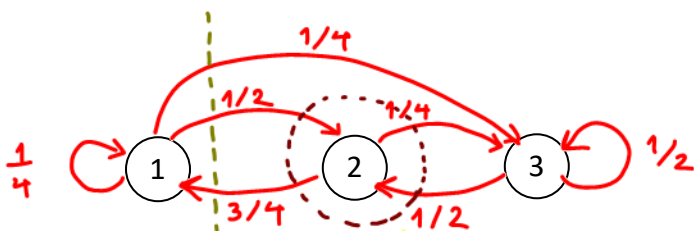
Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Date: <u>10</u> / <u>03</u> /2017			
Name			ID (last 3 digits)
Prapun			5 5 5

Consider a Markov chain whose transition probability matrix is given by $\mathbf{P} = \begin{matrix} & \begin{matrix} \text{to} \\ \text{from} \end{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1/4 & 1/2 & 1/4 \\ 3/4 & 0 & 1/4 \\ 0 & 1/2 & 1/2 \end{bmatrix} \end{matrix}$.

1. Draw the corresponding Markov chain



2. In your drawing above, draw (using a different color) the boundary that corresponds to the balance equation $\frac{1}{2}p_1 - p_2 + \frac{1}{2}p_3 = 0$.

$\Leftrightarrow \frac{1}{2}p_1 + \frac{1}{2}p_3 = p_2 \Rightarrow$ The corresponding boundary is $\text{state } \textcircled{2}$ which is simply the one enclosing state $\textcircled{2}$.
(Note: In the diagram, a dashed circle encloses state 2, and a dashed line encloses states 1 and 3.)

3. Find the steady-state probabilities for this Markov chain. (Don't forget to indicate the boundaries and the corresponding balance equations.)

There are three unknowns: p_1, p_2, p_3 . So, we need three equations. We always have one: $p_1 + p_2 + p_3 = 1 \Rightarrow p_1 + p_3 = 1 - p_2$

Previously, we already have one boundary: $\frac{1}{2}p_1 + \frac{1}{2}p_3 = p_2$

$$1 - p_2 = 2p_2$$

$$p_2 = 1/3$$

We draw another boundary in the Markov chain to get $\frac{1}{4}p_1 + \frac{1}{2}p_1 = \frac{3}{4}p_2$

$$p_1 = p_2 = \frac{1}{3}$$

$$p_3 = 1 - p_1 - p_2 = \frac{1}{3}$$

Therefore, $p_1 = p_2 = p_3 = \frac{1}{3}$