ECS 455: In-Class Exercise # 7

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Date: 0 8 / 0 3 / 2017			
Name	ID (last 3 digits)		
Prapun	5	5	5

1. Consider an evolution of a Markov chain having two states (1 and 2):

   2 → 1 → 2 → 1 → 2 → 1 → 2 → 1 → 1 → 1

   a. Estimate its transition matrix **P**.

   There are 5 transitions from state 1.
   Among those five,
   - two return to state 1
   - three go to state 2

   There are 4 transitions from state 2.
   Among those four,
   - all four go to state 1
   - none returns to state 2

   $$\begin{array}{c} \text{from}\backslash\text{to} \\ 1 \\ 2 \end{array} \begin{bmatrix} 1 & 2 \\ 2/5 & 3/5 \\ 4/4 & 0/4 \end{bmatrix} = \begin{bmatrix} 2/5 & 3/5 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.6 \\ 0 & 1 \end{bmatrix}$$

   b. Let p_i denote the proportion of time that the system spends in state i.
   Estimate both p_1 and p_2.

   The proportion of time that state 1 occurs is $p_1 = \frac{6}{10} = \frac{3}{5} = 0.6$

   The proportion of time that state 2 occurs is $p_2 = \frac{4}{10} = \frac{2}{5} = 0.4$

2. Consider an evolution of a Markov chain having two states (1 and 2):

   2 → 2 → 2 → 2 → 1 → 2 → 2 → 2 → 1 → 1

   a. Estimate its transition matrix **P**.

   There are 2 transitions from state 1.
   Among those five,
   - one returns to state 1
   - one goes to state 2

   There are seven transitions from state 2.
   Among those seven,
   - two go to state 1
   - five return to state 2

   $$\begin{array}{c} \text{from}\backslash\text{to} \\ 1 \\ 2 \end{array} \begin{bmatrix} 1 & 2 \\ 1/2 & 1/2 \\ 2/7 & 5/7 \end{bmatrix} \approx \begin{bmatrix} 0.5 & 0.5 \\ 0.2857 & 0.7143 \end{bmatrix}$$

   b. Let p_i denote the proportion of time that the system spends in state i.
   Estimate both p_1 and p_2.

   The proportion of time that state 1 occurs is $p_1 = \frac{3}{10} = 0.3$

   The proportion of time that state 2 occurs is $p_2 = \frac{7}{10} = 0.7$