

ECS 455: In-Class Exercise #17

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as any of your former groups in this class.
3. Only one submission is needed for each group.
4. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Date: <u>19/05/2017</u>			
Name			ID (last 3 digits)
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Evaluate the following expressions by hand. Show your calculation.

<p style="text-align: center;">$\text{DFT}\{[3 \ -1]\} = [2 \ 4]^T$</p> <p>Let $\vec{x} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$.</p> <p>By definition, $\vec{X} = \text{DFT}\{\vec{x}\} = \Psi_N \vec{x}$ where N is the length of \vec{x}.</p> <p>We've seen in class that $\Psi_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.</p> <p>Hence, $\vec{X} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 3-1 \\ 3+1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$</p>	<p style="text-align: center;">$\text{IDFT}\{[1 \ 0 \ 0]\} = [\frac{1}{3} \ \frac{1}{3} \ \frac{1}{3}]^T$</p> <p>By definition, $\vec{x} = \text{IDFT}\{\vec{X}\} = \frac{1}{N} \Psi_N^* \vec{X} = \frac{1}{N} \Psi_N^* \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$</p> <p style="margin-left: 40px;">$= \frac{1}{N} \times (\text{first column of } \Psi_N^*)$</p> <p style="margin-left: 40px;">$= \frac{1}{3} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$</p> <p>Here, $N=3$</p>
<p style="text-align: center;">$\text{DFT}\{[1 \ 0 \ 0]\} = [1 \ 1 \ 1]^T$</p> <p>First, we apply block-matrix multiplication</p> $\begin{bmatrix} & & \dots & \\ 1 & 1 & \dots & 1 \\ \hline 0 & 0 & \dots & 0 \\ \hline \vdots & \vdots & \dots & \vdots \\ \hline 0 & 0 & \dots & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \sum_{i=1}^n x_i \vec{a}_i$ <p>This implies</p> $\begin{bmatrix} & & \dots & \\ 1 & 1 & \dots & 1 \\ \hline 0 & 0 & \dots & 0 \\ \hline \vdots & \vdots & \dots & \vdots \\ \hline 0 & 0 & \dots & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix} = \vec{a}_1$ <p>Here, $\vec{X} = \Psi_N \vec{x} = \Psi_N \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix}$ $= \text{first column of } \Psi_N$</p> <p style="margin-left: 40px;">$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$</p> <p>Here, $N=3$</p>	<p style="text-align: center;">$\text{DFT}\{[1 \ 0 \ 0 \ 0 \ 0]\} = [1 \ 1 \ 1 \ 1 \ 1]^T$</p> <p style="margin-left: 40px;">$\vec{X} = \Psi_N \vec{x} = \Psi_N \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$</p> <p style="margin-left: 40px;">$= 1^{\text{st}} \text{ column of } \Psi_N$</p> <p style="margin-left: 40px;">$= \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$</p> <p>Here, $N=5$</p>