Sirindhorn International Institute of Technology
Thammasat University at Rangsit
School of Information, Computer and Communication Technology

ECS 455: Problem Set 1

Semester/Year: **2/2016**
Course Title: Mobile Communications
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Course Web Site: http://www2.siit.tu.ac.th/prapun/ecs455/

Due date: February 8, 2017 (Wednesday), 4:30 PM

Instructions

1. (1 pt) Write your first name and the last three digit of your student ID on the upper-right corner of every submitted sheet.
2. (1 pt) For each part, write your explanation/derivation and answer in the space provided.
3. (8 pt) It is important that you try to solve all non-optional problems.
4. Late submission will be heavily penalized.

Questions

1. Cellular communication in the USA was limited by the Federal Communication Commission (FCC) to one of three frequency bands, one around 0.9 GHz, one around 1.9 GHz, and one around 5.8 GHz [Tse and Viswanath, 2005, p. 11]. Find the corresponding **wavelengths**.

Recall that $c = f\lambda$, which means $\lambda = \frac{c}{f}$

Here, $f = 0.9\times10^9$, 1.9×10^9, and 5.8×10^9 Hz.

Hence, $\lambda =$ 33.3, 15.8, and 5.17 cm

2. Under the free-space PL model, how much received power is lost when the operating frequency (carrier frequency) is changed from f_c = 700MHz to f_c = 1,800 MHz? Only the freq f is changed

Friis eqn: $\frac{P_r}{P_t} = \left(\frac{\sqrt{G_{Tx}G_{Rx}}\lambda}{4\pi d}\right)^2 = \left(\frac{\sqrt{G_{Tx}G_{Rx}}}{4\pi d}\frac{c}{f}\right)^2 \Rightarrow P_r = \left(\frac{\sqrt{G_{Tx}G_{Rx}}\, c}{4\pi d}\right) P_t \frac{1}{f^2} \propto \frac{1}{f^2} \Rightarrow \frac{P_{r,new}}{P_{r,old}} = \frac{f_{old}^2}{f_{new}^2}$

In dB, the gain in power is $10\log_{10}\frac{P_{r,new}}{P_{r,old}} = 10\log_{10}\left(\frac{f_{old}}{f_{new}}\right)^2 = 20\log_{10}\frac{f_{old}}{f_{new}} = 20\log_{10}\frac{0.7}{1.8}$

$$P_{r,new}[dB] - P_{r,old}[dB] = -8.2 \text{ dB}.$$

The negative result tells us that the received power is reduced when higher freq. is used.

In conclusion, the loss in received power is 8.2 [dB] (about 87%).

3. Consider a cellular system with operating frequencies around f_c = 900 MHz $= 9\times10^8$ Hz, cells of radius $R =$ 100 m, and *nondirectional* antennas. $\Rightarrow G_{Tx}G_{Rx} = 1$

We need to make sure that the phones at the cell coundary receive at least the min required power

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   a. Under the ***free-space path loss model***, what transmit power is required at the base station such that all mobile devices within the cell receive a minimum power of 10 μW?

By the Friis Equation,

$$\frac{10\mu W}{P_t} = \frac{P_r}{P_t} = \left(\frac{\sqrt{G_{Tx}G_{Rx}}\, c}{4\pi d f}\right)^2 = \left(\frac{\cancel{3}\times \cancel{10^8}}{4\pi\times 100\times \cancel{9}_{3}\times\cancel{10^8}}\right)^2$$

$$P_t = 10\times10^{-6}\times(12\pi\times100)^2 = 144\pi^2\times10^{-1} \approx 142 \text{ W}$$

   b. How does this change if the system frequency is 5 GHz? $= 5\times10^9$ Hz

$$P_t = \frac{P_r}{\left(\frac{\sqrt{G_{Tx}G_{Rx}}\, c}{4\pi d f}\right)^2} = \frac{10\times\cancel{10^{-6}}}{\left(\frac{3\times\cancel{10^8}}{4\pi\times\cancel{100}\times5\times\cancel{10^9}_{10^4}}\right)^2} = \frac{10}{9}\times(4\pi\times5)^2 \approx 4.39 \text{ kW}$$

Alternatively, we can first find the power loss when the freq is changed from 900 MHz to 5GHz. Under the same transmitted power, the received has a loss factor of

$$\left(\frac{0.9}{5}\right)^2.$$

Therefore, to get the same amount of received power, the transmitted power must be increased by a factor of $\left(\frac{5}{0.9}\right)^2$.

[Example 2.1 in Goldsmith, 2005]

So, we need $P_t = \frac{\cancel{144}^{8}\pi^2}{10} \times \frac{\cancel{25}^{5}}{\cancel{81}_{9}}\times 100 = \frac{4000\pi^2}{9} \approx 4.39$ kW

4. Let us take a look at the microwave ultra-wideband (UWB) impulse radio. UWB is a power-limited technology in the unlicensed band of 3.1–10.6 GHz. For the multiband OFDM (MB-OFDM) UWB systems, there are 5 band groups whose centers are provided in Table 1 below

Table 1: Relationship between center frequencies and coverage range for MB-OFDM UWB systems.

Band Group (i)	Center frequency (MHz) (f_i)	Range (meter) ($d_i = \frac{f_1 d_1}{f_i}$)
1	f_1 = 3,960	d_1 = 10
2	f_2 = 5,544	d_2 = 7.14
3	f_3 = 7,128	d_3 = 5.56
4	f_4 = 8,712	d_4 = 4.55
5	f_5 = 10,032	d_5 = 3.95

← See the derivation below

According to the Friis equation, given the same transmitted power, propagation attenuation will be different for each band because they use different frequency. (This variation of received signal strength can be a bothering factor.) If band group 1 can cover 10 meters, estimate the coverage ranges for other band groups in Table 1.

max distance when $f = f_1$.

Show your calculation and explanation here but put your answers in Table 1.

The Friis Equation says $\frac{P_r}{P_t} = \left(\frac{\sqrt{G_{Tx} G_{Rx}}\, c}{4\pi}\right)\left(\frac{1}{df}\right)^2 = \frac{\overset{\text{some constant}}{K}}{d^2 f^2}$

One may immediately conclude from seeing this expression that, to keep the received power constant, we want to maintain the product $d \times f$.

Therefore, we must have $d_1 f_1 = d_i f_i$ which implies $d_i = \frac{d_1 f_1}{f_i}$.

The explanation below tries to look deeper into the reasoning behind this conclusion.

At f_1, the range (max distance) is $d_1 = 10$ m.

So, the min amount of P_r required for the system to work is $P_r = \frac{K}{(d_1 f_1)^2} P_t$.

Now, at f_i, assuming that P_t is the same, then at distance d, the received power is $P_r = \frac{K}{(d f_i)^2} P_t$.

So, to have P_r of at least $\frac{K}{(d_1 f_1)^2} P_t$, we need $\frac{\cancel{K}}{(d f_i)^2}\cancel{P_t} \geq \frac{\cancel{K}}{(d_1 f_1)^2}\cancel{P_t} \Rightarrow d \leq \underbrace{d_1 \frac{f_1}{f_i}}_{\text{so, this is the corresponding range } d_i \text{ (max distance)}} = \frac{39600}{f_i}$

Extra Questions

Here are some optional questions for those who want more practice.

5. Consider the set of empirical measurements of Pr/Pt given in Table 2 below for a system operating around 900 MHz.

Note that in [Nan, Guo, Qiu, Mo, and Takahashi, 2007], the d_i's are incorrectly calculated from $d_i = d_1\left(\frac{f_1}{f_i}\right)^2$ which gives 5.10, 3.09, 2.07, and 1.56 respectively.

Table 2 Path Loss Measurements

Distance from Transmitter	P_r/P_t
10m	-70 dB
20m	-75 dB
50m	-90 dB
100m	-110 dB
300m	-125 dB

a. Find the path loss exponent γ that minimizes the sum of square differences between the calculated dB power ratio using the simplified path loss model and the empirical dB power ratio measurements, assuming that d_0 = 1 m and K is determined from the free space path gain formula at this d_0.

Hint: Recall that $10\log_{10}\frac{P_r}{P_t} = \left(10\log_{10} K\right) + \gamma 10\log_{10}\frac{d_0}{d}$, where $K = \left(\frac{\lambda}{4\pi d_0^2}\right)^2$. The values of *f* and d_0 can be used to determine *K* and hence *b*. So, the only unknown parameter is γ.

Here, we are given 5 values of *d* which we will denote by $d_1, d_2, \ldots, d_5$. For a chosen value of γ, we can plug these distance values into the formula above to calculate $10\log_{10}\frac{P_r}{P_t}$ which will be close to (but not exactly the same as) the $10\log_{10}\frac{P_r}{P_t}$ provided in the table. Our task, then, is to find the best γ that minimize their (average) difference; that is, we want to mimimize

$$\sum_{i=1}^{5}\left(r_i - \left(\left(10\log_{10} K\right) + \gamma 10\log_{10}\frac{d_0}{d_i}\right)\right)^2$$

where r_i is the empirical power ratio measurements provided in the table.

To find the best γ, simply find the root of the derivative wrt. γ.

b. Find the received power at 150 m for the simplified path loss model with the path loss exponent found in part (a) and a transmit power of 1 mW (0 dBm).

[Example 2.3 in Goldsmith, 2005]

6. [Calculus*] Consider the random variable *R* whose $k \ln R \sim \mathcal{N}\left(\mu, \sigma^2\right)$ for some constant μ and positive constants *k* and σ. (An example of this is the random attenuation factor that models the path loss and shadowing effect in wireless communication.)

   a. Find the pdf $f_R\left(r\right)$ of *R*.

   b. Find the expected value $\mathbb{E}R$. (Hint: Let $z = \frac{k\ln\left(r\right) - \mu}{\sigma}$. Then, use the fact that the integration of any pdf will always give 1.)

   c. Find the median of *R*. (This is the value of *r* at which $F_R\left(r\right) = \frac{1}{2}$.)

Solution for Q5: Simplified PL Model: From Measurements

(a) Simplified path loss model: $\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}$

In dB, this is $P_r[dB] - P_t[dB] = \underbrace{10\log_{10} K}_{b} + \gamma \underbrace{10\log_{10}\left(\frac{d_0}{d}\right)}_{\alpha}$ ★

For free-space path gain, $K = \left(\frac{\lambda}{4\pi d_0}\right)^2 = \left(\frac{c}{4\pi d_0 f}\right)^2$

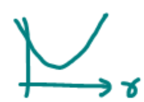
Here, $f = 900$ MHz, $d_0 = 1$ m

Therefore, $\underbrace{10\log_{10} K}_{b} = 10\log_{10}\left(\frac{c}{4\pi d_0 f}\right)^2 \approx -31.53$ dB

Note that ★ is of the form $y(\alpha) = b + \gamma\alpha$

We are given five pairs of $\underset{r_i}{y_i}, \underset{10\log_{10}\left(\frac{d_0}{d_i}\right)}{\alpha_i}$.

We want to find γ that **minimizes** $MSE = \sum_{i=1}^{5}\left(y(\alpha_i) - y_i\right)^2 = \underbrace{\sum_i\left(b + \gamma\alpha_i - y_i\right)^2}_{\text{parabola wrt. } \gamma}$

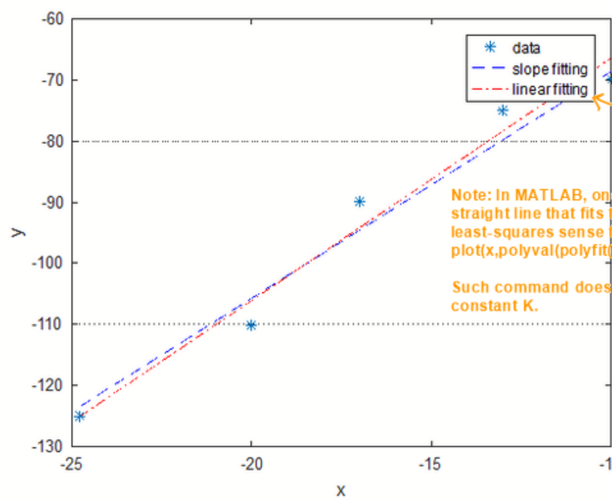


So, we find $\frac{d}{d\gamma}MSE = \sum_i 2\left(b + \gamma\alpha_i - y_i\right)\alpha_i = b\sum_i\alpha_i + \gamma\sum_i\alpha_i^2 - \sum_i\alpha_i y_i$

The value of γ that makes $\frac{d}{d\gamma}MSE = 0$ is

$$\gamma = \frac{\overbrace{\sum_i \alpha_i y_i}^{\approx 8.50\times10^3} - b\overbrace{\sum_i \alpha_i}^{\approx -84.77}}{\underset{\approx 1.57\times10^3}{\sum_i \alpha_i^2}} \approx 3.71$$

y_i	d_i	α_i
-70	10	-10
-75	20	-13
-90	50	-17
-110	100	-20
-125	300	-24.77



Note: In MATLAB, one can find and plot the straight line that fits the data best in a least-squares sense from plot(x,polyval(polyfit(x,y,1),x))

Such command does not utilize the known constant K.

(b) At $d = 150$ m, $\alpha = 10\log_{10}\left(\frac{d_0}{d}\right) \approx -21.76$.

$\Downarrow$

$y = b + \gamma\alpha \approx -112.24$

So, $P_r[dB] - P_t[dB] = -112.24$

$P_r[dB] = P_t[dB] - 112.24$

$P_r[dBm] = \underbrace{P_t[dBm]}_{0} - 112.24 = -112.24$ dBm.

In this problem, we apply the technique studied in ECS315 to find the pdf of a function of continuous RV.

Solution for Q6 Shadowing: Lognormal RV

(a) Let $X = k \ln R$. Then, $X \sim \mathcal{N}(\mu, \sigma^2)$ which implies $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$.

First, we will show that R is a continuous RV.

From $X = k\ln R$, we see that for each $R = r$ value, there is at most one real-valued $X = x$ that satisfies their relationship. Because X is a continuous RV, we can then conclude that R is also a continuous RV.

Next, to find the pdf of R, we first find its CDF and then take derivative.

From $\underbrace{R = e^{X/k}}$,

① This means $R > 0$.

So, $F_R(r) = 0$ for $r \le 0$.

$\Rightarrow \frac{d}{dr} F_R(r) = 0$ for $r < 0$

② For $r > 0$

$$F_R(r) = P[R \le r] = P\left[e^{X/k} \le r\right] \overset{\text{for } r>0}{=} P[X \le k \ln r] = F_X(k \ln r)$$

$$f_R(r) = \frac{d}{dr} F_R(r) \overset{\text{for } r>0}{=} \frac{k}{r} f_X(k\ln r) = \frac{k}{r}\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{k\ln r - \mu}{\sigma}\right)^2}$$

So, combining ① and ②, we have

$$f_R(r) = \begin{cases} \frac{k}{r}\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{k\ln r-\mu}{\sigma}\right)^2}, & r > 0, \\ 0, & r \le 0. \end{cases}$$

Although we did not calculate the value of $f_R(r)$ @ $r=0$, because R is a continuous RV, we can simply assign $f_R(r) = 0$ @ $r = 0$.

(b) $$\mathbb{E}R = \int_0^\infty r f_R(r) = \int_0^\infty \frac{k}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{k\ln r - \mu}{\sigma}\right)^2} dr$$

Let $x = \frac{k\ln r - \mu}{\sigma} \Rightarrow dx = \frac{k}{\sigma}\frac{1}{r} dr$

$\Downarrow$

$dr = \frac{\sigma}{k} r\, dx = \frac{\sigma}{k} e^{\frac{\sigma x + \mu}{k}} dx$

Therefore,

$$\mathbb{E}R = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{\sigma}{k}x} e^{\frac{\mu}{k}} e^{-\frac{1}{2}x^2} dx = e^{\mu/k} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(x^2 - 2\frac{\sigma}{k}x + \left(\frac{\sigma}{k}\right)^2\right)} e^{\frac{1}{2}\left(\frac{\sigma}{k}\right)^2} dx$$

$-\infty$ ← as $r \to 0$ we have $x \to -\infty$

$$= e^{\frac{\mu}{k} + \frac{1}{2}\left(\frac{\sigma}{k}\right)^2} \underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(x - \frac{\sigma}{k}\right)^2} dx} = e^{\frac{\mu}{k} + \frac{1}{2}\left(\frac{\sigma}{k}\right)^2}$$

↳ This is the pdf of a Gaussian RV whose expected value is σ/k and variance is 1. Integrating pdf gives 1.

(c) We want to find r at which $F_R(r) = \frac{1}{2}$.

Recall, from part (a), that $F_R(r) = F_X(k\ln r)$.

By the symmetry of $f_X(x)$ around its expected value μ, we know that $F_X(k\ln r) = \frac{1}{2}$ when $k\ln r = \mu$.

Therefore, our sought-after value of r is $e^{\mu/k}$.