Sirindhorn International Institute of Technology
Thammasat University at Rangsit
School of Information, Computer and Communication Technology

ECS 455: Problem Set 7

Semester/Year: **2/2014**
Course Title: Mobile Communications
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Due date: Not due

1. Consider the list of Walsh sequence of order 64 provided in [Lee and Miller, 1998, Table 5.2].

Table 5.2 Walsh functions of order 64 (indexed by zero crossings)

W_{0}	0000000000000000	0000000000000000	0000000000000000	0000000000000000
W_{1}	0000000000000000	0000000000000000	1111111111111111	1111111111111111
W_{2}	0000000000000000	1111111111111111	1111111111111111	0000000000000000
W_{3}	0000000000000000	1111111111111111	0000000000000000	1111111111111111
W_{4}	0000000011111111	1111111100000000	0000000011111111	1111111100000000
W_{5}	0000000011111111	1111111100000000	1111111100000000	0000000011111111
W_{6}	0000000011111111	0000000011111111	1111111100000000	1111111100000000
W_{7}	0000000011111111	0000000011111111	0000000011111111	0000000011111111
W_{8}	0000111111110000	0000111111110000	0000111111110000	0000111111110000
W_{9}	0000111111110000	0000111111110000	1111000000001111	1111000000001111
W_{10}	0000111111110000	1111000000001111	1111000000001111	0000111111110000
W_{11}	0000111111110000	1111000000001111	0000111111110000	1111000000001111
W_{12}	0000111100001111	1111000011110000	0000111100001111	1111000011110000
W_{13}	0000111100001111	1111000011110000	1111000011110000	0000111100001111
W_{14}	0000111100001111	0000111100001111	1111000011110000	1111000011110000
W_{15}	0000111100001111	0000111100001111	0000111100001111	0000111100001111
W_{16}	0011110000111100	0011110000111100	0011110000111100	0011110000111100
W_{17}	0011110000111100	0011110000111100	1100001111000011	1100001111000011
W_{18}	0011110000111100	1100001111000011	1100001111000011	0011110000111100
W_{19}	0011110000111100	1100001111000011	0011110000111100	1100001111000011
W_{20}	0011110011000011	1100001100111100	0011110011000011	1100001100111100
W_{21}	0011110011000011	1100001100111100	1100001100111100	0011110011000011
W_{22}	0011110011000011	0011110011000011	1100001100111100	1100001100111100
W_{23}	0011110011000011	0011110011000011	0011110011000011	0011110011000011
W_{24}	0011001111001100	0011001111001100	0011001111001100	0011001111001100
W_{25}	0011001111001100	0011001111001100	1100110000110011	1100110000110011
W_{26}	0011001111001100	1100110000110011	1100110000110011	0011001111001100
W_{27}	0011001111001100	1100110000110011	0011001111001100	1100110000110011
W_{28}	0011001100110011	1100110011001100	0011001100110011	1100110011001100
W_{29}	0011001100110011	1100110011001100	1100110011001100	0011001100110011
W_{30}	0011001100110011	0011001100110011	1100110011001100	1100110011001100
W_{31}	0011001100110011	0011001100110011	0011001100110011	0011001100110011
W_{32}	0110011001100110	0110011001100110	0110011001100110	0110011001100110
W_{33}	0110011001100110	0110011001100110	1001100110011001	1001100110011001
W_{34}	0110011001100110	1001100110011001	1001100110011001	0110011001100110
W_{35}	0110011001100110	1001100110011001	0110011001100110	1001100110011001
W_{36}	0110011010011001	1001100101100110	0110011010011001	1001100101100110
W_{37}	0110011010011001	1001100101100110	1001100101100110	0110011010011001
W_{38}	0110011010011001	0110011010011001	1001100101100110	1001100101100110
W_{39}	0110011010011001	0110011010011001	0110011010011001	0110011010011001
W_{40}	0110100110010110	0110100110010110	0110100110010110	0110100110010110
W_{41}	0110100110010110	0110100110010110	1001011001101001	1001011001101001
W_{43}	0110100110010110	1001011001101001	0110100110010110	1001011001101001
W_{44}	0110100101101001	1001011010010110	0110100101101001	1001011010010110
W_{45}	0110100101101001	1001011010010110	1001011010010110	0110100101101001
W_{46}	0110100101101001	0110100101101001	1001011010010110	1001011010010110
W_{47}	0110100101101001	0110100101101001	0110100101101001	0110100101101001
W_{48}	0101101001011010	0101101001011010	0101101001011010	0101101001011010
W_{49}	0101101001011010	0101101001011010	1010010110100101	1010010110100101
W_{50}	0101101001011010	1010010110100101	1010010110100101	0101101001011010
W_{51}	0101101001011010	1010010110100101	0101101001011010	1010010110100101
W_{52}	0101101010100101	1010010101011010	0101101010100101	1010010101011010
W_{53}	0101101010100101	1010010101011010	1010010101011010	0101101010100101
W_{54}	0101101010100101	0101101010100101	1010010101011010	1010010101011010
W_{55}	0101101010100101	0101101010100101	0101101010100101	0101101010100101
W_{56}	0101010110101010	0101010110101010	0101010110101010	0101010110101010
W_{57}	0101010110101010	0101010110101010	1010101001010101	1010101001010101
W_{58}	0101010110101010	1010101001010101	1010101001010101	0101010110101010
W_{59}	0101010110101010	1010101001010101	0101010110101010	1010101001010101
W_{60}	0101010101010101	1010101010101010	0101010101010101	1010101010101010
W_{61}	0101010101010101	1010101010101010	1010101010101010	0101010101010101
W_{62}	0101010101010101	0101010101010101	1010101010101010	1010101010101010
W_{63}	0101010101010101	0101010101010101	0101010101010101	0101010101010101

In class, we observed that one of the sequenced is missing.
Find the content of that sequence.
Hint: Use MATLAB.

2. Select the terms (provided at the end of the problem) to complete the following description of OFDM systems:

   Wireless systems suffer from ______________ problem. Equalization can be used to mitigate this problem. Another important technique that works effectively in wireless systems is OFDM. The general idea is to ______________ the symbol or bit time so that it is ______________ compared with the channel delay spread. To do this, we separate the original data stream into multiple parallel substreams and transmit the substreams via different carrier frequencies, creating parallel subchannels. This is called ______________. In such direct implementation, there are two new problems to solve: bandwidth inefficiency and complexity of the transceivers. The inefficient use of bandwidth is caused by the need of ______________ between adjacent subchannels. Bandwidth efficiency can be improved by utilizing ______________. The computational complexity of the transceivers is solved by the use of ______________.

   Here are the terms to use. Some term(s) is/are not used.

   - FFT and IFFT
   - FDM
   - multipath fading
   - local oscillators
   - guard bands
   - guard times
   - reduce
   - increase
   - small
   - large
   - spectral efficiency
   - orthogonality

3. Evaluate the following expressions **by hand**. Show your calculation. (You may use MATLAB to check your answers later.)
   a. $\text{DFT}\{[3 \quad -1]\}$
   b. $\text{DFT}\{[1 \quad 0 \quad 0]\}$
   c. $\text{IDFT}\{[1 \quad 0 \quad 0]\}$
   d. $\text{DFT}\{[1 \quad 0 \quad 0 \quad 0 \quad 0]\}$
   e. $[1 \quad 2 \quad -1] * [2 \quad 1 \quad -2]$
   f. $[1 \quad 2 \quad -1] \circledast [2 \quad 1 \quad -2]$
   g. $[1 \quad 2 \quad -1 \quad 0] \circledast [2 \quad 1 \quad -2 \quad 0]$
   h. $[1 \quad 2 \quad -1 \quad 0 \quad 0] \circledast [2 \quad 1 \quad -2 \quad 0 \quad 0]$

4. In this question, we will consider an OFDM system in discrete time. The channel is characterized by $\mathbf{h} = \begin{bmatrix} 2 & -1 \end{bmatrix}$. We would like to transmit $\mathbf{S} = \begin{bmatrix} 1 & -1 & 2 & 1 & -1 & 2 & 1 & 2 \end{bmatrix}$ of data across this channel using OFDM. For simplicity, we will assume that there is no noise. Let N = 4 be the length of each OFDM symbol.
   a. Find the transmitted vector **x**. (Apply IFFT with scaling by $\sqrt{N}$. Then add cyclic prefix.) To reduce the overhead, the cyclic prefix should be as short as possible.
   b. The received vector is $\mathbf{y} = \mathbf{x} * \mathbf{h}$. (Note that this is a regular convolution.) Find $\mathbf{y}$.
   c. Find $\mathbf{H}$ which is the FFT of the zero-padded $\mathbf{h}$.
   d. Remove the "irrelevant parts" from $\mathbf{y}$. Then apply FFT with scaling by $1/\sqrt{N}$. Finally, use the corresponding property in frequency domain of the circular convolution (in time) for DFT to recover the original data $\mathbf{S}$ from $\mathbf{y}$.
5. Recall that the baseband OFDM modulated signal can be expressed as

$$s(t) = \sum_{k=0}^{N-1} S_k \frac{1}{\sqrt{N}} 1_{[0,T_s]}(t) \exp\left(j \frac{2\pi kt}{T_s} \right)$$

where $S_0, S_1, \ldots, S_{N-1}$ are the (potentially complex-valued) messages.

Let $T_s = 1$ [ms], N = 8, and

$$(S_0, S_1, \ldots, S_{N-1}) = (1-j, 1+j, 1, 1-j, -1-j, 1, 1-j, -1+j)$$

   a. Use MATLAB `ifft` command to plot $\mathrm{Re}\{s(t)\}$ for $0 \le t \le T_s$.
   Hint: Use oversampling with large value of L.
   b. Let
      i. $a(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \mathrm{Re}\{S_k\} \cos\left(\frac{2\pi kt}{T_s} \right)$
      ii. $b(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \mathrm{Im}\{S_k\} \sin\left(\frac{2\pi kt}{T_s} \right)$

   What is the relationship between $a(t)$, $b(t)$, and $\mathrm{Re}\{s(t)\}$?
   c. Let

$$s_2(t) = \sum_{k=0}^{N-1} S_k^* \frac{1}{\sqrt{N}} 1_{[0,T_s]}(t) \exp\left(j \frac{2\pi kt}{T_s} \right).$$

   Note the extra conjugation in $s_2(t)$.
   What is the relationship between $a(t)$, $b(t)$, and $\mathrm{Re}\{s_2(t)\}$?
   d. Use MATLAB to plot $a(t)$ and $b(t)$ for $0 \le t \le T_s$.
   Use the relationships found in parts (b) and (c).