Sirindhorn International Institute of Technology
Thammasat University at Rangsit
School of Information, Computer and Communication Technology

ECS 455: Problem Set 5

Semester/Year: **2/2014**
Course Title: Mobile Communications
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Course Web Site: http://www2.siit.tu.ac.th/prapun/ecs455/

Due date: April 10, 2015 (Friday), 10:30AM

Instructions

1. It is important that you try to solve all problems. (5 pt)
   For questions (or parts of questions) that require the use of MATLAB, include printouts of your scripts and the results from the command window.
2. ONE sub-question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work carefully on all of them.
3. Late submission will be heavily penalized.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Questions

1. (**Markov Chain**) "The Land of Oz is blessed by many things, but not by good weather. They never have two nice days in a row. If they have a nice day, they are just as likely to have snow as to have rain on the next day. If they have snow or rain, they have an even chance of having the same the next day. If there is change from snow or rain, only half of the time is this a change to a nice day." [Grinstead and Snell, Ex 11.1][Kemeny, Snell, and Thompson, 1974]
   a. Draw the Markov chain corresponding to how the weather in the Land of Oz changes from one day to the next.
      Hint: This Markov chain will have three states: rain (R), nice (N), and snow (S).
   b. Find the corresponding transition probability matrix **P**.

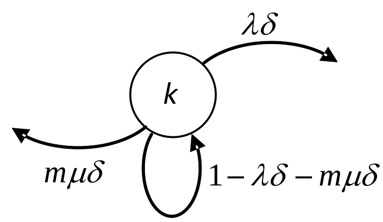
c. Find the steady-state probabilities
    i. by using balance equations
    ii. by using the eigen-values & eigen-vectors (in MATLAB)

d. Modify the script MarkovChain_Demo1.m discussed in class to check your answers in part (b) using simulation in MATLAB.

e. Suppose it is snowing in the Land of Oz today.
    i. (*) **Calculate** the chance that it will be a nice day tomorrow.
    ii. (*) **Calculate** the chance that it will be a nice day the day after tomorrow.
    iii. **Estimate** the chance that it will be a nice day next year (365 days later).

2. Consider a system which has 3 channels. We would like to find the blocking probability via the Markov chain method. For each of the following models, **draw the Markov chain** via discrete time approximation. Find (1) the **steady-state probabilities** and (2) the long-term **call blocking probability**.
    a. ***Erlang B*** model: Assume that the *total* call request rate is 10 calls per hour and the average call duration is 12 mins.
    b. ***Engset*** model: Assume that there are 5 users. The call request rate for *each* user is 2 calls per hour and the average call duration is 12 mins.
    c. ***Engset*** model: Assume that there are 100 users. The call request rate for *each* user is 0.1 calls per hour and the average call duration is 12 mins.

3. Consider another modification of the M/M/m/m (Erlang B) system. (There are infinite users) Assume that there is a queue that can be used to hold all requested call which cannot be immediately assigned a channel. This is referred to as an M/M/m/∞ or simply M/M/m system. We will define state k as the state where there are k calls in the system. If $k \le m$, then all of these calls are ongoing. If $k > m$, then m of them are ongoing and k-m of them are waiting in the queue.
   Assume that the total call request process is Poisson with rate λ and that the call durations are i.i.d. exponential random variables with expected value $1/\mu$.
   Also assume that $\frac{\lambda}{\mu} < m$.
    a. **Draw the Markov chain** via discrete time approximation. Don't forget to indicate the transition probabilities (in terms of λ, μ, and δ) on the arrows.
       Hint: there are infinite number of states. The transition probabilities for state k which is < m are the same as in the M/M/m/m system. For $k \ge m$, the transition probabilities are given below:



Explain why the above transition probabilities make sense.

b. Find the **steady-state probabilities** using balance equations

c. Find the long-term **delayed call probability** (the probability that a call request occurs when all *m* channels are busy and thus has to wait).
Hint: This will be a summation of many steady-state probabilities. When you simplify your answer, the final answer should be

$$\frac{A^m}{A^m + m!\left(1-\frac{A}{m}\right)\sum_{k=0}^{m-1}\frac{A^k}{k!}}.$$

4. (Optional) In class we have seen that the steady-state probabilities for the Engset model are given by

$$p_i = \frac{\binom{n}{i}A_u^i}{\sum_{k=0}^{m}\binom{n}{k}A_u^k} = \frac{\binom{n}{i}A_u^i}{z(m,n)},\ 0 \le i \le m,$$

where $z(m,n) = \sum_{k=0}^{m}\binom{n}{k}A^k$.

a. Express p_m (time congestion) in the form $p_m = 1 - \frac{z(m-c,n)}{z(m,n)}$.

What is the value of c?
Hint: c is an integer.

b. The blocked call probability is given by $P_b = \frac{(n-m)\binom{n}{m}A_u^m}{\sum_{k=0}^{m}\binom{n}{k}(n-k)A_u^k}$ which can be rewritten in the form $P_b = 1 - \frac{z(m-c_1, n-c_2)}{z(m-c_3, n-c_4)}$.

Find c_1, c_2, c_3, c_4.
Hint: c_1, c_2, c_3, c_4 are all integrers.

c. Suppose $m = n - 1$. Simplify the expression for P_b.

Hint: Your answer should be of the form $\left(g\left(A_u\right)\right)^m$ for some function *g* of A_u.