# Sirindhorn International Institute of Technology <br> Thammasat University at Rangsit 

School of Information, Computer and Communication Technology

## ECS 455: Problem Set 4

Semester/Year: 2/2014
Course Title: Mobile Communications
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Course Web Site: http://www2.siit.tu.ac.th/prapun/ecs455/

## Due date: Not Due

## Questions

1) Consider a single base station mobile radio system. Suppose the total call request rate is three calls per hour, each call lasting an average of 5 minutes. Assume $M / \mathrm{M} / \mathrm{m} / \mathrm{m}$ model.
a) What is the probability that there are exactly two call requests during the time interval from 9 AM to 11 AM ?
b) Consider two time intervals:
$I_{1}=$ the time interval from 9 AM to 11 AM , and
$\mathrm{I}_{2}=$ the time interval from 1 PM to 3 PM.
i) Find the probability that there are exactly two call requests in $I_{1}$ and there are exactly three call requests in $I_{2}$.
ii) Suppose we know that there are exactly two call requests in $I_{1}$. Find the probability that there are exactly three call requests in $\mathrm{I}_{2}$.
c) Suppose we know that a call request happens at 10 AM .
i) Suppose there is an available channel for this call.
(1) Find the probability that it is still ongoing at 11 AM.
(2) Find the probability that it ends before 10:05 AM.
(3) Suppose the call is still ongoing at 10:05 AM. What is the probability that it is still ongoing at 10:06AM?
ii) Find the probability that the next call request happens before 11AM.
2) Complete the following $\mathrm{M} / \mathrm{M} / \mathrm{m} / \mathrm{m}$ description with the following terms:
(I) Bernoulli
(IV) Gaussian
(II) binomial
(III) exponential
(V) geometric
(VI) Poisson

The Erlang B formula is derived under some assumptions. Two important assumptions are (1) the call request process is modeled by a/an
$\qquad$ (A) $\qquad$ process and (2) the call durations are assumed to be i.i.d.
$\qquad$ (B) $\qquad$ random variables. For the call request process, the times
between adjacent call requests can be shown to be i.i.d. $\qquad$ (C)
random variables. On the other hand, if we consider non-overlapping time intervals, the numbers of call requests in these intervals are
$\qquad$ (D) $\qquad$ random variables.

In order to analyze or simulate the system described above, we consider slotted time where the duration of each time slot is small. This technique shifts our focus from continuous-time Markov chain to discrete-time Markov chain. In the limit, for the call request process, only one of the two events can happen during any particular slot: either (1) there is one new call request or (2) there is no new call request. When the slots are small and have equal length, the numbers of new call requests in the slots can be approximated by i.i.d. $\qquad$ (E) $\qquad$ random variables. In which case, if we count the total number of call requests during $n$ slots, we will get a/an
$\qquad$ (F) $\qquad$ random variable because it is a sum of i.i.d.
$\qquad$ (E) $\qquad$ random variables.

When we consider a particular time interval $I$ (not necessarily small), the number of slots in this interval will increase as the slots get smaller. In the limit, the number of call requests in the time interval $I$ which we approximated by a $\qquad$ (F) $\qquad$ random variable before will approach a/an
$\qquad$ (D) $\qquad$ random variable.

Similarly, if we consider the numbers of slots between adjacent call requests, these number will be i.i.d. $\qquad$ (G) $\qquad$ random variables. These random variables can be thought of as discrete counterparts of the i.i.d.
$\qquad$ (C) $\qquad$ random variables in the continuous-time model.

