Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

ECS 455: Problem Set 3

Semester/Year: **2/2014**

Course Title: Mobile Communications

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Course Web Site: http://www2.siit.tu.ac.th/prapun/ecs455/

Due date: February 27, 2015 (Friday), 10:30AM

Instructions

1. Solve all problems. (5 pt)
2. ONE sub-question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work carefully on all of them.
3. It is important that you try to solve all problems. (5 pt)
4. Late submission will be heavily penalized.
5. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
6. You may use the MATLAB code from

   http://infohost.nmt.edu/~borchers/erlang.html

   to evaluate the Erlang B formula. Figure 3.6 from [Rappaport, 2002], which is included here in Appendix A, may give a rough approximation.

Questions

1. **[HW3 2014/2 Q1]** Some possible values of cluster size is N = 1, 3, 4, or 7. Find the next fifteen lowest values of N.

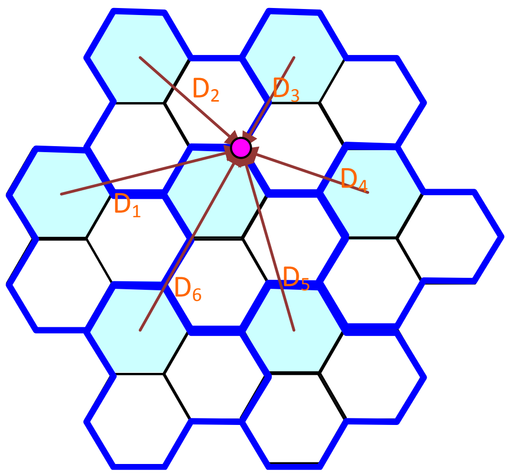
2. **[HW3 2014/2 Q2]** 20 MHz of total spectrum is allocated for a **duplex** wireless cellular system. Suppose each **simplex** channel uses 25 kHz RF bandwidth.
   a. Find the number of **duplex** channels.
   b. Find the total number of duplex channels per cell, if N = 4 cell reuse is used.

3. **[HW3 2014/2 Q3]** In this question, we will find the unsectored (using omnidirectional antennas) SIR value when N = 3.

(a) Recall that the SIR can be calculated from

$$\frac{S}{I} = \frac{kR^{-\gamma}}{\sum_{i=1}^{K} kD_i^{-\gamma}}$$

where D_i is the distance from the *i*th interfering co-channel cell base station. Find all the distance D_i in the figure below.



Express them as a function of *R* (the distance from the center of the hexagon to its vertex.) Hint: $D_1 = \sqrt{13}R$.

(b) Calculate the SIR (in dB) using the D_i values that you got from part (a). Assume a path loss exponent of $\gamma = 4$.

(c) Approximate all value of D_i by the **center-to-center** distance *D* between the nearest co-channel. Express *D* as a function of *R*. Recalculate the SIR (in dB) with all D_i replaced by D.

(d) Compare your answers from part (b) and part (c).

4. **[HW3 2014/2 Q4]** A cellular service provider decides to use a digital TDMA scheme which can tolerate a signal-to-interference ratio of 15 dB in the worst case. Find the optimal value of *N* for (a) omnidirectional antennas, (b) 120° sectoring, and (c) 60° sectoring. (Assume a path loss exponent of $\gamma = 4$.)

Hint: Approximate the SIR value by

$$\frac{S}{I} = \frac{kR^{-\gamma}}{K \times \left(kD^{-\gamma}\right)} = \frac{1}{K}\left(\frac{D}{R}\right)^{\gamma} = \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

where *K* is the number of (first-tier) interfering (co-channel) base stations.

5. **[HW3 2014/2 Q5]** How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?
   (a) 5
   (b) 15
   (c) 25
   Assume each user generates 0.1 Erlangs of traffic.

6. **[HW3 2014/2 Q6]** Assume each user of a single base station mobile radio system averages three calls per
   hour, each call lasting an average of 5 minutes.
   (a) What is the traffic intensity for each user?
   (b) Find the number of users that could use the system with 1% blocking if only one channel is available.
   (c) Find the number of users that could use the system with 1% blocking if five trunked channels are available.
   (d) If the number of users you found in (c) is suddenly doubled, what is the new blocking probability of the five channel trunked mobile radio system?

Extra Question

7. **[HW3 2014/2 Q7]** Reflection from a ground plane: Consider the propagation model in Figure 1 where there is a reflected path from the ground plane. Let r_1 be the length of the direct path. Let r_2 be the length of the reflected path (summing the path length from the transmitter to the ground plane and the path length from the ground plane to the receiver). [Tse and Viswanath, 2005, Section 2.1.5 and Exercise 2.5]

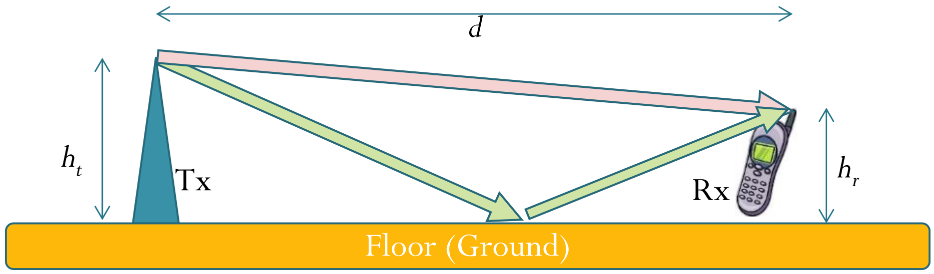


Figure 1: Illustration of a direct path and a reflected path off a ground plane [Tse and Viswanath, 2005, Figure 2.6].

a. The plot in Figure **2** shows $r_2 - r_1$ when $h_t = 50$ m and $h_r = 2$ m. Note that the plot is done in log-log scale.

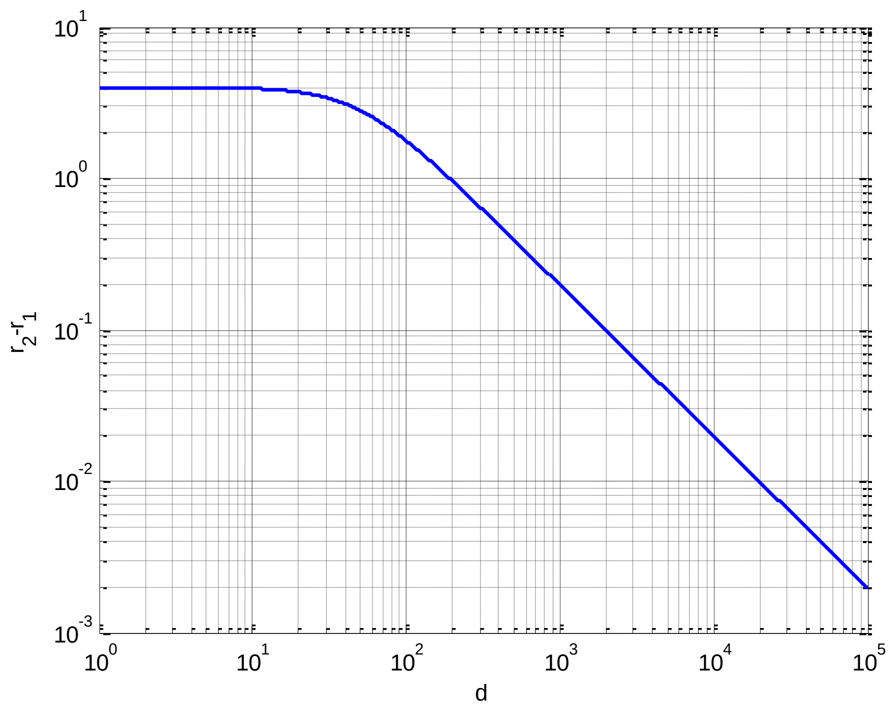


Figure 2: Log-log plot of r_2-r_1.

i. Use MATLAB to plot a similar graph for the case when $h_t = 100$ m and $h_r = 1$ m.

ii. From the plot in Figure **2** and the plot from the previous part, notice that when *d* is large, the graphs becomes straight lines. Estimate the slope of the line and then use the slope to show that $r_2 - r_1 \propto \frac{1}{d}$ when *d* is large.

b. Now, instead of looking at the plot(s), analytically derive the fact that when the horizontal distance *d* between the antennas becomes very large relative to their vertical displacements (h_t and h_r) from the ground, $r_2 - r_1 \propto \frac{1}{d}$.

Hint: For small *x*, $\sqrt{1+x} \approx 1 + \frac{x}{2}$.

Remark: This will also allow you to find the proportionality constant.

c. In class, we have shown that when the transmitted signal is $x(t) = \sqrt{2P_t}\cos(2\pi f_c t)$, the received signal is given by

$$y(t) = \frac{\alpha}{r_1}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r_1}{c}\right)\right) - \frac{\alpha}{r_2}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r_2}{c}\right)\right).$$

i. Show that $\frac{P_y}{P_x} = \left|\frac{\alpha}{r_1}e^{-j2\pi f_c\frac{r_1}{c}} - \frac{\alpha}{r_2}e^{-j2\pi f_c\frac{r_2}{c}}\right|^2$.

Hint: Use phasor form to show that

for $g(t) = a_1\cos(2\pi f_c t + \phi_1) + a_2\cos(2\pi f_c t + \phi_2)$, we have

$P_g = \frac{1}{2}\left|a_1 e^{j\phi_1} + a_2 e^{j\phi_2}\right|^2$.

ii. Justify each equality/approximation below:

$$\frac{P_y}{P_x} \overset{(1)}{=} \left|\frac{\alpha}{r_1} - \frac{\alpha}{r_2}e^{-j2\pi f_c\frac{r_2-r_1}{c}}\right|^2 \overset{(2)}{\approx} \left|\frac{\alpha}{r_1} - \frac{\alpha}{r_2}e^{-j2\pi\frac{2h_t h_r}{\lambda d}}\right|^2 \overset{(3)}{\approx} \left|\frac{\alpha}{d}\left(1 - e^{-j\frac{4\pi h_t h_r}{\lambda d}}\right)\right|^2$$

$$\overset{(4)}{\approx} \left(\frac{\alpha}{d}\right)^2\left|1 - \left(1 - j\frac{4\pi h_t h_r}{\lambda d}\right)\right|^2 \overset{(5)}{=} \frac{\alpha^2}{d^2}\left|j2\pi\frac{2h_t h_r}{\lambda d}\right|^2 = \left(\frac{4\pi\alpha h_t h_r}{\lambda d^2}\right)^2$$

Appendix A

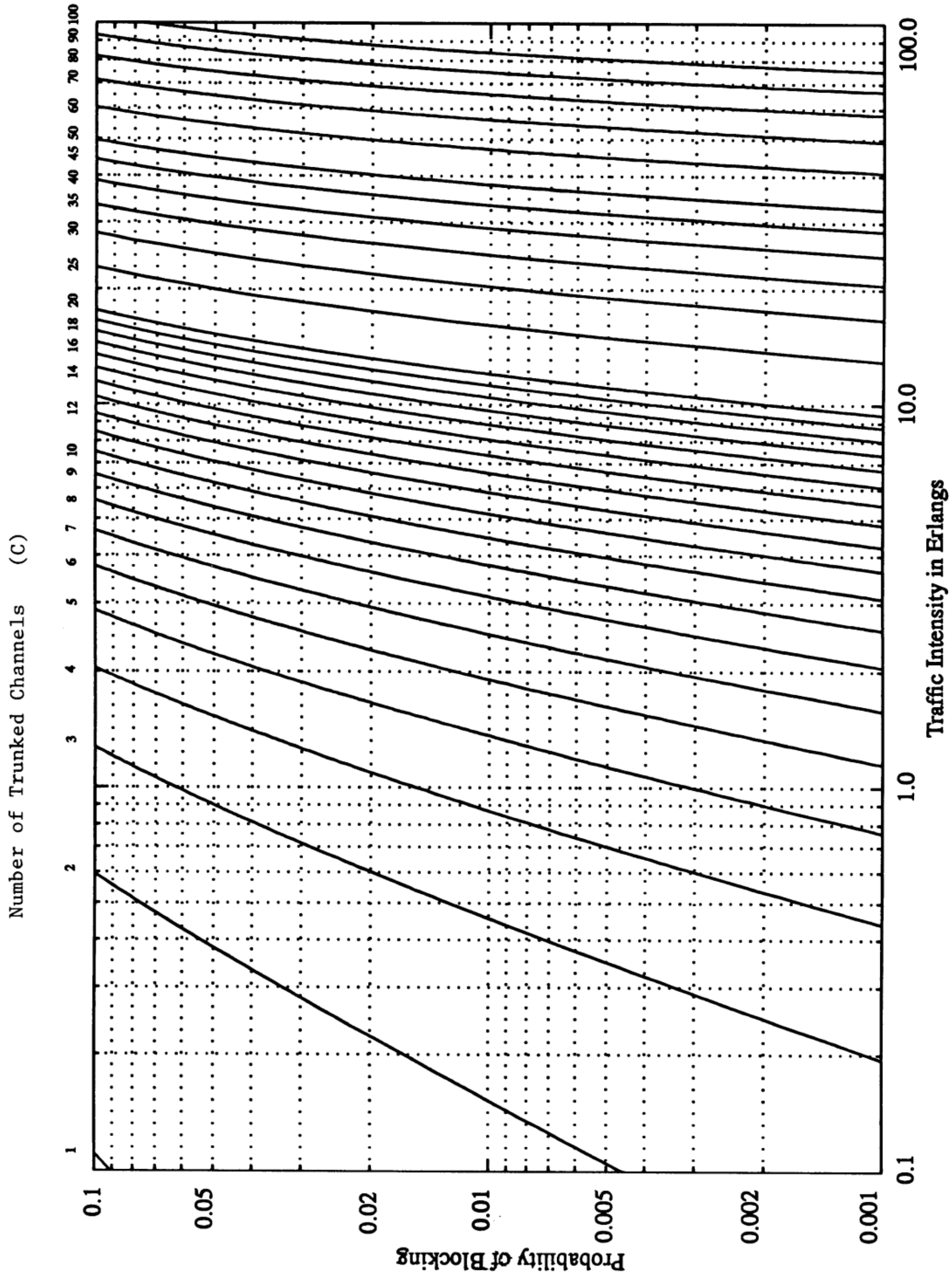


Figure 3.6 The Erlang B chart showing the probability of blocking as functions of the number of channels and traffic intensity in Erlangs.