Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

ECS 455: Problem Set 2

Semester/Year: **2/2014**

Course Title: Mobile Communications

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Course Web Site: http://www2.siit.tu.ac.th/prapun/ecs455/

Due date: February 6, 2015 (Friday), 10:30AM

Instructions

1. Solve all problems. (5 pt)
2. ONE sub-question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work carefully on all of them.
3. Late submission will be heavily penalized.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Questions

1. **[HW2 2014/2 Q1]** Cellular communication in the USA was limited by the Federal Communication Commission (FCC) to one of three frequency bands, one around 0.9 GHz, one around 1.9 GHz, and one around 5.8 GHz [Tse and Viswanath, 2005, p. 11]. Find the corresponding **wavelengths**.

2. **[HW2 2014/2 Q2]** Under the free-space PL model, how much received power is lost when the operating frequency (carrier frequency) is changed from f_c = 700MHz to f_c = 1,800 MHz?

3. **[HW2 2014/2 Q3]** Consider a cellular system with operating frequencies around f_c = 900 MHz, cells of radius 100 m, and *nondirectional* antennas.

a. Under the ***free-space path loss model***, what transmit power is required at the base station such that all mobile devices within the cell receive a minimum power of 10 μW?

b. How does this change if the system frequency is 5 GHz?

[Example 2.1 in Goldsmith, 2005]

4. **[HW2 2014/2 Q4]** Let us take a look at the microwave ultra-wideband (UWB) impulse radio. UWB is a power-limited technology in the unlicensed band of 3.1–10.6 GHz. For the multiband OFDM (MB-OFDM) UWB systems, there are 5 band groups whose centers are provided in Table 1 below

Table 1: Relationship between center frequencies and coverage range for MB-OFDM UWB systems.

Band Group	Center frequency (MHz)	Range (meter)
1	3,960	10
2	5,544	?
3	7,128	?
4	8,712	?
5	10,032	?

According to the Friis equation, given the same transmitted power, propagation attenuation will be different for each band because they use different frequency. (This variation of received signal strength can be a bothering factor.) If band group 1 can cover 10 meters, estimate the coverage ranges for other band groups in Table 1.

5. **[HW2 2014/2 Q5]** (Power Calculation) For each of the following signals $g(t)$, find (i) its corresponding power $P_g = \left\langle |g(t)|^2 \right\rangle$, (ii) the power $P_x = \left\langle |x(t)|^2 \right\rangle$ where $x(t) = g(t)\cos(10t)$, and (iii) the power $P_y = \left\langle |y(t)|^2 \right\rangle$ where $y(t) = g(t)\cos(50t)$.

a. $g(t) = 3\cos(10t + 30^\circ)$.

b. $g(t) = 3\cos(10t + 30^\circ) + 4\cos(10t + 120^\circ)$. (Hint. First, use phasor form to combine the two components into one sinusoid.)

c. $g(t) = 3\cos(10t) + 3\cos(10t + 120^\circ) + 3\cos(10t + 240^\circ)$

Extra Questions

Here are some questions for those who want extra practice.

6. **[HW2 2014/2 Q6]** Consider the set of empirical measurements of Pr/Pt given in Table 2 below for a system operating around 900 MHz.

Table 2 Path Loss Measurements

Distance from Transmitter	P_r/P_t
10m	-70 dB
20m	-75 dB
50m	-90 dB
100m	-110 dB
300m	-125 dB

a. Find the path loss exponent γ that minimizes the sum of square differences between the calculated dB power ratio using the simplified path loss model and the empirical dB power ratio measurements, assuming that d_0 = 1 m and K is determined from the free space path gain formula at this d_0.

Hint: Recall that

$$10\log_{10}\frac{P_r}{P_t}=\left(10\log_{10}K\right)+\gamma 10\log_{10}\frac{d_0}{d},$$

where $K=\left(\frac{\lambda}{4\pi d_0^2}\right)^2$. The values of *f* and d_0 can be used to determine *K* and hence *b*. So, the only unknown parameter is γ.

Here, we are given 5 values of *d* which we will denote by $d_1, d_2, \ldots, d_5$. For a chosen value of γ, we can plug these distance values into the formula above to calculate $10\log_{10}\frac{P_r}{P_t}$ which will be close to (but not exactly the same as) the $10\log_{10}\frac{P_r}{P_t}$ provided in the table. Our task, then, is to find the best γ that minimize their (average) difference; that is, we want to mimimize

$$\sum_{i=1}^{5}\left(r_i-\left(\left(10\log_{10}K\right)+\gamma 10\log_{10}\frac{d_0}{d_i}\right)\right)^2$$

where r_i is the empirical power ratio measurements provided in the table.

To find the best γ, simply find the root of the derivative wrt. γ.

b. Find the received power at 150 m for the simplified path loss model with the path loss exponent found in part (a) and a transmit power of 1 mW (0 dBm).

[Example 2.3 in Goldsmith, 2005]

7. **[HW2 2014/2 Q7]** [Calculus*] Consider the random variable *R* whose $k \ln R \sim \mathcal{N}\left(\mu, \sigma^2\right)$ for some constant μ and positive constants *k* and σ. (An example of this is the random attenuation factor that models the path loss and shadowing effect in wireless communication.)

   a. Find the pdf $f_R(r)$ of *R*.

   b. Find the expected value $\mathbb{E}R$. (Hint: Let $z = \frac{k \ln(r) - \mu}{\sigma}$. Then, use the fact that the integration of any pdf will always give 1.)

   c. Find the median of *R*. (This is the value of *r* at which $F_R(r) = \frac{1}{2}$.)

8. **[HW2 2014/2 Q8]** Recall that $\langle x(t) \rangle = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t) dt$.

   a. Suppose $x(t)$ is periodic with period T_0. Show that $\langle x(t) \rangle = \frac{1}{T_0} \int_{T_0} x(t) dt$ where the integration is over any interval of length T_0.

   b. Show that $\left\langle e^{j\beta t} \right\rangle = \begin{cases} 1, & \beta = 0, \\ 0, & \beta \neq 0. \end{cases}$

   c. Consider a signal $g(t) = \sum_{k=1}^{n} c_k e^{j2\pi f_k t}$ where the f_k's are distinct.

   Show that $P_g = \sum_{k=1}^{n} |c_k|^2$.

   Remark: Because f_k is not always of the form kf_0 for some fundamental frequency f_0, we cannot simply use Parseval's theorem for Fourier series to conclude that $P_g = \sum_{k=1}^{n} |c_k|^2$.

Hint: Recall that $P_g = \left\langle |g(t)|^2 \right\rangle$. Write $|g(t)|^2 = g(t)g^*(t)$. Use this to express $|g(t)|^2$ as a double sum. You can now find the power with the help of your answer from part (a).