ECS 455: Mobile Communications **2014/2**

HW 1 — Due: January 30, 10:30 AM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. [1] Using `MATLAB` to find the magnitude spectrum[2] of a signal:

A signal $g(t)$ can often be expressed in analytical form as a function of time t, and the Fourier transform is defined as the integral of $g(t)\exp(-j2\pi ft)$. Often however, there is no analytical expression for a signal, that is, there is no (known) equation that represents the value of the signal over time. Instead, the signal is defined by measurements of some physical process. For instance, the signal might be the waveform at the input to the receiver, the output of a linear filter, or a sound waveform encoded as an mp3 file.

In all these cases, it is not possible to find the spectrum by analytically performing a Fourier transform. Rather, the discrete Fourier transform (or DFT, and its cousin, the more rapidly computable fast Fourier transform, or FFT) can be used to find the spectrum or frequency content of a measured signal. The `MATLAB` function `plotspec.m`, which plots the spectrum of a signal can be downloaded from our course website. Its help file[3] notes

```
% plotspec(x,Ts) plots the spectrum of the signal x
% Ts = time (in seconds) between adjacent samples in x
```

[1]Based on [Johnson, Sethares, and Klein, 2011, Sec 3.1 and Q3.3].

[2]also referred to by "amplitude spectrum" or simply "spectrum"

[3]You can view the help file for the `MATLAB` function `xxx` by typing `help xxx` at the `MATLAB` prompt. If you get an error such as `xxx` not found, then this means either that the function does not exist, or that it needs to be moved into `MATLAB`'s search path.

(a) The function `plotspec.m` is easy to use. For instance, the spectrum of a rectangular pulse[4] $g(t) = 1[0 \le t \le 2]$ can be found using:

```
% specrect.m plot the spectrum of a square wave
close all
time=20;                    % length of time
Ts=1/100;                   % time interval between samples
t=0:Ts:(time-Ts);           % create a time vector
x=[t ≤ 2];                 % rectangular pulse 1[0 ≤ t ≤ 2]
plotspec(x,Ts)              % call plotspec to draw spectrum
xlim([-5,5])
```

The output of `specrect.m` is shown in Figure 1.1. The top plot shows the first 20 seconds of $g(t)$. The bottom plot shows $|G(f)|$.

Now, use what we learn in class about the Fourier transform of a rectangular pulse (and the time-shift property) to find a simplified expression for $|G(f)|$. Does your expression agree with the bottom plot in Figure 1.1.

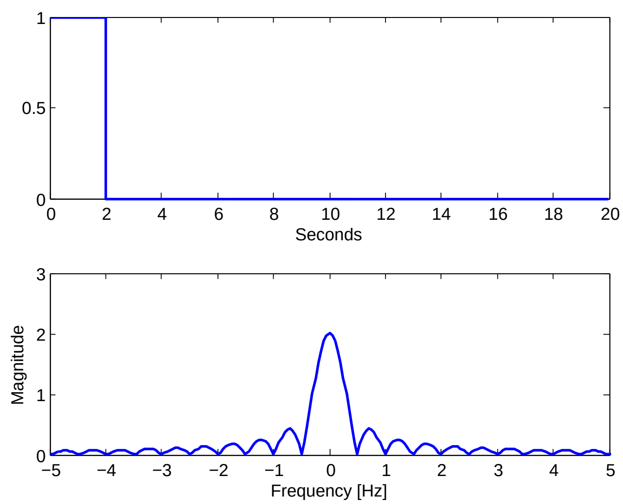

Figure 1.1: Plots from `specrect.m`

[4]Here, we define a rectangular pulse using the indicator function $1[\cdot]$. This function outputs a 1 when the statement inside the square brackets is true; otherwise, it outputs a 0. For example,

$$1\left[0 \le t \le 2\right] = \begin{cases} 1, & 0 \le t \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

(b) Modify the code in `specrect.m` to find the (magnitude) spectrum of an exponential pulse

$$s(t) = e^{-t}u(t).$$

Note that you may want to change the parameter `time` to capture most of the content of $s(t)$ in the time domain. You may also use the command `xlim` to "zoom in" the spectrum plot.

(c) Continue from part (b), find $S(f)$ analytically. Compare your analytical answer with the plot in part (b).

(d) `MATLAB` can also perform symbolic manipulation when symbolic toolbox is installed. Run the file `SymbFourier.m`. Check whether you have the same result as part (c).

Problem 2. [5]

(a) Consider the cosine pulse

$$p(t) = \begin{cases} \cos(10\pi t), & -1 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

(i) Use `MATLAB` to plot $p(t)$ for $-3 \le t \le 3$.

(ii) Find $P(f)$ analytically.

(iii) Use the expression from part (ii) to plot $P(f)$ in `MATLAB`.

(b) Consider the cosine pulse

$$p(t) = \begin{cases} \cos(10\pi t), & 2 \le t \le 4 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find $P(f)$ analytically.

(ii) Use `MATLAB`. Mimic the code in `specrect.m` to plot the spectrum of $p(t)$.

(iii) Compare your analytical answer from part (i) with the plot in part (ii).

[5] Inspired by [Carlson and Crilly, 2009, Q2.2-1 and Q2.2-2].

Problem 3. You are given the baseband signals (i) $m(t) = \cos 1000\pi t$; (ii) $m(t) = 2\cos 1000\pi t + \cos 2000\pi t$; (iii) $m(t) = (\cos 1000\pi t) \times (\cos 3000\pi t)$. For each one, do the following.

(a) Sketch the spectrum of $m(t)$.

(b) Sketch the spectrum of the DSB-SC signal $m(t)\cos 10,000\pi t$.

[Lathi and Ding, 2009, Q4.2-1]

Extra Question

Here is an optional question for those who want more practice.

Problem 4. Use properties of Fourier transform to evaluate the following integrals. (Do not integrate directly. Recall that $\text{sinc}(x) = \frac{\sin(x)}{x}$.) Clearly state the property or properties that you use.

(a) $\int\limits_{-\infty}^{\infty} \text{sinc}\left(\sqrt{5}x\right) dx$

(b) $\int\limits_{-\infty}^{\infty} e^{-2\pi f \times 2j} 2\text{sinc}\left(2\pi f\right) \left(e^{-2\pi f \times 5j} 2\text{sinc}\left(2\pi f\right)\right)^* df$

(c) $\int\limits_{-\infty}^{\infty} \text{sinc}\left(\sqrt{5}x\right) \text{sinc}\left(\sqrt{7}x\right) dx$

(d) $\int\limits_{-\infty}^{\infty} \text{sinc}\left(\pi\left(x-5\right)\right) \text{sinc}\left(\pi\left(x-\frac{7}{2}\right)\right) dx$