Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

ECS 455: Problem Set 5

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Course Title: Mobile Communications

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1. Consider the list of Walsh sequence of order 64 provided in [Lee and Miller, 1998, Table 5.2].

Table 5.2 Walsh functions of order 64 (indexed by zero crossings)

W_0	0000000000000000	0000000000000000	0000000000000000	0000000000000000
W_1	0000000000000000	0000000000000000	1111111111111111	1111111111111111
W_2	0000000000000000	1111111111111111	1111111111111111	0000000000000000
W_3	0000000000000000	1111111111111111	0000000000000000	1111111111111111
W_4	0000000011111111	1111111100000000	0000000011111111	1111111100000000
W_5	0000000011111111	1111111100000000	1111111100000000	0000000011111111
W_6	0000000011111111	0000000011111111	1111111100000000	1111111100000000
W_7	0000000011111111	0000000011111111	0000000011111111	0000000011111111
W_8	0000111111110000	0000111111110000	0000111111110000	0000111111110000
W_9	0000111111110000	0000111111110000	1111000000001111	1111000000001111
W_{10}	0000111111110000	1111000000001111	1111000000001111	0000111111110000
W_{11}	0000111111110000	1111000000001111	0000111111110000	1111000000001111
W_{12}	0000111100001111	1111000011110000	0000111100001111	1111000011110000
W_{13}	0000111100001111	1111000011110000	1111000011110000	0000111100001111
W_{14}	0000111100001111	0000111100001111	1111000011110000	1111000011110000
W_{15}	0000111100001111	0000111100001111	0000111100001111	0000111100001111
W_{16}	0011110000111100	0011110000111100	0011110000111100	0011110000111100
W_{17}	0011110000111100	0011110000111100	1100001111000011	1100001111000011
W_{18}	0011110000111100	1100001111000011	1100001111000011	0011110000111100
W_{19}	0011110000111100	1100001111000011	0011110000111100	1100001111000011
W_{20}	0011110011000011	1100001100111100	0011110011000011	1100001100111100
W_{21}	0011110011000011	1100001100111100	1100001100111100	0011110011000011
W_{22}	0011110011000011	0011110011000011	1100001100111100	1100001100111100
W_{23}	0011110011000011	0011110011000011	0011110011000011	0011110011000011
W_{24}	0011001111001100	0011001111001100	0011001111001100	0011001111001100
W_{25}	0011001111001100	0011001111001100	1100110000110011	1100110000110011
W_{26}	0011001111001100	1100110000110011	1100110000110011	0011001111001100
W_{27}	0011001111001100	1100110000110011	0011001111001100	1100110000110011
W_{28}	0011001100110011	1100110011001100	0011001100110011	1100110011001100
W_{29}	0011001100110011	1100110011001100	1100110011001100	0011001100110011
W_{30}	0011001100110011	0011001100110011	1100110011001100	1100110011001100
W_{31}	0011001100110011	0011001100110011	0011001100110011	0011001100110011
W_{32}	0110011001100110	0110011001100110	0110011001100110	0110011001100110
W_{33}	0110011001100110	0110011001100110	1001100110011001	1001100110011001
W_{34}	0110011001100110	1001100110011001	1001100110011001	0110011001100110
W_{35}	0110011001100110	1001100110011001	0110011001100110	1001100110011001
W_{36}	0110011010011001	1001100101100110	0110011010011001	1001100101100110
W_{37}	0110011010011001	1001100101100110	1001100101100110	0110011010011001
W_{38}	0110011010011001	0110011010011001	1001100101100110	1001100101100110
W_{39}	0110011010011001	0110011010011001	0110011010011001	0110011010011001
W_{40}	0110100110010110	0110100110010110	0110100110010110	0110100110010110
W_{41}	0110100110010110	0110100110010110	1001011001101001	1001011001101001
W_{43}	0110100110010110	1001011001101001	0110100110010110	1001011001101001
W_{44}	0110100101101001	1001011010010110	0110100101101001	1001011010010110
W_{45}	0110100101101001	1001011010010110	1001011010010110	0110100101101001
W_{46}	0110100101101001	0110100101101001	1001011010010110	1001011010010110
W_{47}	0110100101101001	0110100101101001	0110100101101001	0110100101101001
W_{48}	0101101001011010	0101101001011010	0101101001011010	0101101001011010
W_{49}	0101101001011010	0101101001011010	1010010110100101	1010010110100101
W_{50}	0101101001011010	1010010110100101	1010010110100101	0101101001011010
W_{51}	0101101001011010	1010010110100101	0101101001011010	1010010110100101
W_{52}	0101101010100101	1010010101011010	0101101010100101	1010010101011010
W_{53}	0101101010100101	1010010101011010	1010010101011010	0101101010100101
W_{54}	0101101010100101	0101101010100101	1010010101011010	1010010101011010
W_{55}	0101101010100101	0101101010100101	0101101010100101	0101101010100101
W_{56}	0101010110101010	0101010110101010	0101010110101010	0101010110101010
W_{57}	0101010110101010	0101010110101010	1010101001010101	1010101001010101
W_{58}	0101010110101010	1010101001010101	1010101001010101	0101010110101010
W_{59}	0101010110101010	1010101001010101	0101010110101010	1010101001010101
W_{60}	0101010101010101	1010101010101010	0101010101010101	1010101010101010
W_{61}	0101010101010101	1010101010101010	1010101010101010	0101010101010101
W_{62}	0101010101010101	0101010101010101	1010101010101010	1010101010101010
W_{63}	0101010101010101	0101010101010101	0101010101010101	0101010101010101

In class, we observed that one of the sequenced is missing.

Find the content of that sequence.

Hint: Use MATLAB.

2. Select the terms (provided at the end of the problem) to complete the following description of OFDM systems:

   Wireless systems suffer from ______________ problem. Equalization can be used to mitigate this problem. Another important technique that works effectively in wireless systems is OFDM. The general idea is to ______________ the symbol or bit time so that it is ______________ compared with the channel delay spread. To do this, we separate the original data stream into multiple parallel substreams and transmit the substreams via different carrier frequencies, creating parallel subchannels. This is called ______________. In such direct implementation, there are two new problems to solve: bandwidth inefficiency and complexity of the transceivers. The inefficient use of bandwidth is caused by the need of ______________ between adjacent subchannels. Bandwidth efficiency can be improved by utilizing ______________. The computational complexity of the transceivers is solved by the use of ______________.

   Here are the terms to use. Some term(s) is/are not used.

   - FFT and IFFT
   - FDM
   - multipath fading
   - local oscillators
   - guard bands
   - guard times
   - reduce
   - increase
   - small
   - large
   - spectral efficiency
   - orthogonality

3. Recall that the baseband OFDM modulated signal can be expressed as

$$s(t) = \sum_{k=0}^{N-1} S_k \frac{1}{\sqrt{N}} 1_{[0,T_s]}(t) \exp\left(j \frac{2\pi kt}{T_s} \right)$$

   where $S_0, S_1, \ldots, S_{N-1}$ are the (potentially complex-valued) messages.
   Let $T_s = 1$ [ms], *N* = 8, and

$$(S_0, S_1, \ldots, S_{N-1}) = (1-j, 1+j, 1, 1-j, -1-j, 1, 1-j, -1+j)$$

   a. Use MATLAB `ifft` command to plot $\mathrm{Re}\{s(t)\}$ for $0 \le t \le T_s$.
   b. Let
      i. $a(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \mathrm{Re}\{S_k\} \cos\left(\frac{2\pi kt}{T_s} \right)$
      ii. $b(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \mathrm{Im}\{S_k\} \sin\left(\frac{2\pi kt}{T_s} \right)$

What is the relationship between $a(t)$, $b(t)$, and $\operatorname{Re}\{s(t)\}$?

c. Let

$$s_2(t) = \sum_{k=0}^{N-1} S_k^* \frac{1}{\sqrt{N}} 1_{[0,T_s]}(t) \exp\left(j \frac{2\pi kt}{T_s} \right).$$

Note the extra conjugation in $s_2(t)$.

What is the relationship between $a(t)$, $b(t)$, and $\operatorname{Re}\{s_2(t)\}$?

d. Use MATLAB to plot $a(t)$ and $b(t)$ for $0 \le t \le T_s$.

4. Evaluate the following expressions by hand. Show your calculation.
   a. $\mathrm{DFT}\{[3 \quad -1]\}$
   b. $\mathrm{DFT}\{[1 \quad 0 \quad 0]\}$
   c. $\mathrm{IDFT}\{[1 \quad 0 \quad 0]\}$
   d. $\mathrm{DFT}\{[1 \quad 0 \quad 0 \quad 0 \quad 0]\}$
   e. $[1 \quad 2 \quad -1] * [2 \quad 1 \quad -2]$
   f. $[1 \quad 2 \quad -1] \circledast [2 \quad 1 \quad -2]$
   g. $[1 \quad 2 \quad -1 \quad 0] \circledast [2 \quad 1 \quad -2 \quad 0]$
   h. $[1 \quad 2 \quad -1 \quad 0 \quad 0] \circledast [2 \quad 1 \quad -2 \quad 0 \quad 0]$

5. Consider the discrete-time complex FIR channel model

$$y[n] = \{h * x\}[n] + w[n] = \sum_{m=0}^{2} h[m] x[n-m] + w[n]$$

where $w[n]$ is zero-mean additive Gaussian noise.

In this question, assume that *h*[*n*] has unit energy and that $H(z)$ has two zeros at

$z_1 = \rho e^{j\frac{2\pi}{3}}$ and $z_2 = \frac{1}{\rho}$ where $\rho < 1$.

a. The information given above implies that

$$h[k] = \frac{1}{\sqrt{E_{h_{un}}}} h_{un}[k] \text{ and } H(z) = \frac{1}{\sqrt{E_{h_{un}}}} H_{un}(z)$$

where

$$h_{un}[k] = \delta[k] - (z_1 + z_2)\delta[k-1] + z_1 z_2 \delta[k-2],$$

$$H_{un}(z) = (1 - z_1 z^{-1})(1 - z_2 z^{-1}) = 1 - (z_1 + z_2) z^{-1} + z_1 z_2 z^{-2},$$

and

$$E_{h_{un}} = 1^2 + |z_1 + z_2|^2 + |z_1|^2 |z_2|^2 .$$

Plot $\left|H\left(e^{j\omega}\right)\right| = \left|H(z)\big|_{z=e^{j\omega}}\right|$ in the range $\omega = 0 : \frac{2\pi}{80} : 2\pi$ for ρ = 0.5 and 0.99.

See Appendix A for the discussion on how I get $h[k]$ and $H(z)$.

b. For OFDM system with block size *N* = 8, find the corresponding channel gains $H_k = H(z)\big|_{z=e^{j\frac{2\pi}{N}k}}$, *k* = 0, 1, 2, …, *N*-1 for ρ = 0.5 and 0.99. In particular, complete the following table.

Ch # *k*	$\rho = 0.5$		$\rho = 0.99$	
	H_k	$\lvert H_k\rvert$	H_k	$\lvert H_k\rvert$
0	-0.5455 + 0.1890i	0.5774	-0.0087 + 0.0050i	0.0100
1				
2				
3				
4	0.9820 + 0.5669i	1.1339	0.5860 + 0.9949i	1.1547
5				
6				
7				

6. OFDM simulation: Write a MATLAB code to perform the following operations
   a. Generate 10,000 OFDM blocks, each is an 8 dimensional QPSK vector. Each element of the vector is independently and randomly chosen from the constellation set $M = \{1+j, 1-j, -1+j, -1-j\}$.
   b. Perform the MATLAB's IFFT to each vector. Multiply the result by $\sqrt{N}$. (See Q3a for the reason why we need to multiply by $\sqrt{N}$.)
   c. Add the cyclic prefix to each block and transmit over the FIR channel defined in the previous question. Assume $w[n] \equiv 0$. Consider two cases: ρ = 0.5 and 0.99.
   d. At the receiver, remove the cyclic prefix and perform the FFT to get R_k. Scale the results by $1/\sqrt{N}$.
   e. Assume that the receiver knows H_k. **Detect** the transmitted symbols at each channel.

When there is no noise, you have $R_k = H_k S_k$. Therefore, $S_k = \frac{R_k}{H_k}$. **<u>Record</u>** the symbol error rates (SER) <u>for each channel</u>.

Remark: All of them should be 0 in this question. The goal of this problem is to make you that you understand the OFDM system enough to write down MATLAB code for it.

7. (Difficult) Repeat Question 6. However, in this question, the channel noise is generally non-zero. $w[n]$ is now i.i.d. complex-valued Gaussian noise. Its real and imaginary parts are i.i.d. zero-mean Gaussian with variance $N_0/2$ where $N_0 = 1$.

   For the decoder, use the ML (maximum likelihood) detector to decode back S_k. To do this, first calculate $\frac{R_k}{H_k}$. Then, find $\hat{S}_k$ which is defined to be the *<u>closest</u>* message *s* in the constellation to $\frac{R_k}{H_k}$.

   a. Complete the following table. The H_k can be copied from the Table in Q2b.

	$\rho = 0.5$		$\rho = 0.99$	
Ch # k	$\lvert H_k \rvert$	SER	$\lvert H_k \rvert$	SER
0				
1				
2				
3				
4				
5				
6				
7				

   Note that a symbol is counted as decoded correctly if both real and imaginary parts are decoded correctly.

   b. What is the relationship between $|H_k|$ and SER.

Appendix A

The channel *h* has two zeros at $z_1 = \rho e^{j\frac{2\pi}{3}}$ and $z_2 = \frac{1}{\rho}$.

Thus, before normalized to unit energy, we have

$$H_{un}(z) = \left(1 - z_1 z^{-1}\right)\left(1 - z_2 z^{-1}\right) = 1 - \left(z_1 + z_2\right) z^{-1} + z_1 z_2 z^{-2}.$$

Inverse Z-transform gives

$$h_{un}[k] = \delta[k] - \left(z_1 + z_2\right)\delta[k-1] + z_1 z_2 \delta[k-2].$$

The energy of $h_{un}[k]$ is $E_{h_{un}} = 1^2 + \left|z_1 + z_2\right|^2 + \left|z_1\right|^2 \left|z_2\right|^2$.

So,

$$h[k] = \frac{1}{\sqrt{E_{h_{un}}}} h_{un}[k] \text{ and } H(z) = \frac{1}{\sqrt{E_{h_{un}}}} H_{un}(z).$$

Appendix B

When there is some zero-mean Gaussian noise, $R_k = H_k S_k + W_k$ where W_k is the noise (in the frequency domain). Therefore,

$$\frac{R_k}{H_k} = S_k + \underbrace{\frac{W_k}{H_k}}_{\text{New Noise}}.$$

Because the noise is Gaussian and zero-mean, the noise will most-likely not take $\frac{R_k}{H_k}$ too far from S_k. Therefore, the ML detector gives

$$\hat{S}_k = \arg\min_{s \in M} \left\{ \left\| R_k - s H_k \right\| \right\} = \arg\min_{s \in M} \left\{ \left\| \frac{R_k}{H_k} - s \right\| \right\},$$

i.e. it detects S_k as the <u>*closest*</u> message *s* in the constellation to $\frac{R_k}{H_k}$. Of course, the noise can be large and shift $\frac{R_k}{H_k}$ too far from the original S_k. Therefore, $\hat{S}_k$ may be different from S_k. This is when symbol error occurs.