Sirindhorn International Institute of Technology

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School of Information, Computer and Communication Technology

# ECS 455: Problem Set 5 Solution

**Semester/Year:** **2/2011**
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1. In MATLAB, it is easy to generate the Hadamard matrix by the command `hadamard(N)`. However, note that the Walsh sequences are indexed by zero crossings. We can follow the recipe for index changing provided in lecture or we can directly generate the Walsh matrix by the command `ifwht(eye(N))`.

   Here, `N = 64`. So, we use `ifwh(eye(64))`. Note however that the outputs will be in the {1,-1} from. To map them back to the {0,1} form, we calculate

   `W = (1- ifwht(eye(64)))/2.`

   Finally, the missing row is $W_{42}$. Note that the first row is $W_0$. Therefore, $W_{42}$ is the 43rd row. Its content is given by `W(43,:)`. From MATLAB, $W_{42}$ is

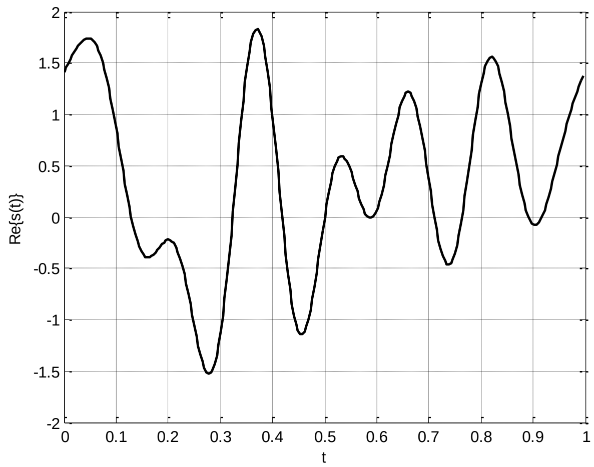
   [0110 1001 1001 0110 1001 0110 0110 1001
   1001 0110 0110 1001 0110 1001 1001 0110]

2. Wireless systems suffer from multipath fading problem. Equalization can be used to mitigate this problem. Another important technique that works effectively in wireless systems is OFDM. The general idea is to increase the symbol or bit time so that it is large compared with the channel delay spread. To do this, we separate the original data stream into multiple parallel substreams and transmit the substreams via different carrier frequencies, creating parallel subchannels. This is called FDM. In such direct implementation, there are two new problems to solve: bandwidth inefficiency and complexity of the transceivers. The inefficient use of bandwidth is caused by the need of guard bands between adjacent subchannels. Bandwidth efficiency can be improved by

utilizing orthogonality. The computational complexity of the transceivers is solved by the use of FFT and IFFT.

3. **Solution**

   a.




   b. $\mathrm{Re}\{s(t)\} = a(t) - b(t)$

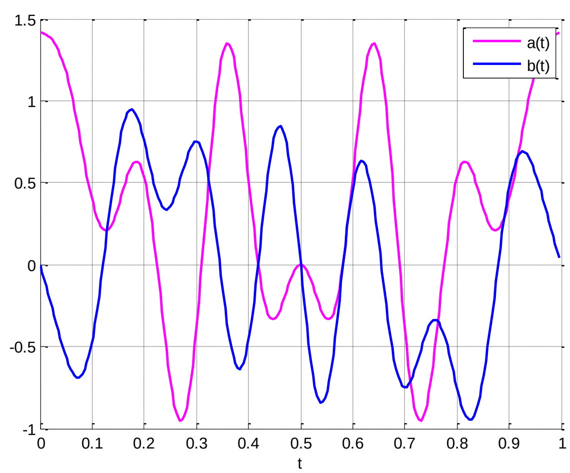
   c. $\mathrm{Re}\{s_2(t)\} = a(t) + b(t)$

   d. From part (b) and (c), we have

$$a(t) = \frac{\mathrm{Re}\{s_2(t)\} + \mathrm{Re}\{s(t)\}}{2}$$

and

$$b(t) = \frac{\mathrm{Re}\{s_2(t)\} - \mathrm{Re}\{s(t)\}}{2}.$$



4.

a. $\text{DFT}\{[3 \quad -1]\} = [2 \quad 4]$

b. $\text{DFT}\{[1 \quad 0 \quad 0]\} = [1 \quad 1 \quad 1]$

c. $\text{IDFT}\{[1 \quad 0 \quad 0]\} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

d. $\text{DFT}\{[1 \quad 0 \quad 0 \quad 0 \quad 0]\} = [1 \quad 1 \quad 1 \quad 1 \quad 1]$

e. $[1 \quad 2 \quad -1] * [2 \quad 1 \quad -2] = [2\,5\, -2\, -5\, 2]$

f. $[1 \quad 2 \quad -1] \circledast [2 \quad 1 \quad -2] = [-3\, 7\, -2]$

g. $[1 \quad 2 \quad -1 \quad 0] \circledast [2 \quad 1 \quad -2 \quad 0] = [4\, 5\, -2\, -5]$

h. $[1 \quad 2 \quad -1 \quad 0 \quad 0] \circledast [2 \quad 1 \quad -2 \quad 0 \quad 0] = [2\, 5\, -2\, -5\, 2]$

5. Consider the discrete-time complex FIR channel model

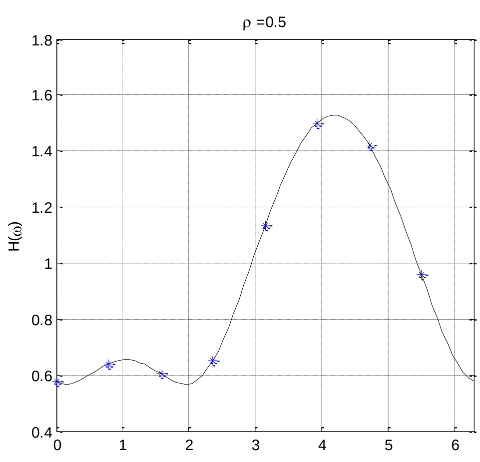
$$y[n] = \{h * x\}[n] + w[n] = \sum_{m=0}^{2} h[m]x[n-m] + w[n]$$

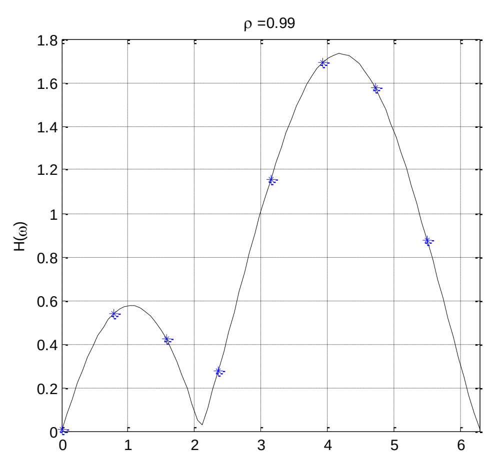
where $w[n]$ is zero-mean additive Gaussian noise.

In this question, assume that *h*[*n*] has unit energy and that $H(z)$ has two zeros at $z_1 = \rho e^{j\frac{2\pi}{3}}$ and $z_2 = \frac{1}{\rho}$ where $\rho < 1$.

**Solution**

a. The plots of $\left|H\left(e^{j\omega}\right)\right| = \left|H(z)\right|_{z=e^{j\omega}}\right|$ in the range $0 \le \omega \le 2\pi$ for ρ = 0.5 and 0.99 are shown below:



b. For OFDM system with block size $N$ = 8, find the corresponding channel gains $H_k = H(z)\big|_{z=e^{j\frac{2\pi}{N}k}}$, $k$ = 0, 1, 2, …, $N$-1 for ρ = 0.5 and 0.99.

| Ch # $k$ | $\rho = 0.5$ | | $\rho = 0.99$ | |
|---|---|---|---|---|
| | $H\left(e^{j\frac{2\pi}{N}k}\right)$ | $\left\lvert H\left(e^{j\frac{2\pi}{N}k}\right)\right\rvert$ | $H\left(e^{j\frac{2\pi}{N}k}\right)$ | $\left\lvert H\left(e^{j\frac{2\pi}{N}k}\right)\right\rvert$ |
| 0 | -0.5455 + 0.1890i | 0.5774 | -0.0087 + 0.0050i | 0.0100 |
| 1 | 0.1407 + 0.6246i | 0.6403 | 0.5170 + 0.1489i | 0.5380 |
| 2 | 0.4657 + 0.3858i | 0.6047 | 0.3710 - 0.2026i | 0.4227 |
| 3 | 0.4649 + 0.4555i | 0.6508 | -0.0624 + 0.2716i | 0.2787 |
| 4 | 0.9820 + 0.5669i | 1.1339 | 0.5860 + 0.9949i | 1.1547 |
| 5 | 1.4881 - 0.1882i | 1.4999 | 1.6375 + 0.4284i | 1.6927 |
| 6 | 0.8436 - 1.1417i | 1.4196 | 1.3609 - 0.7973i | 1.5773 |
| 7 | -0.3480 - 0.8919i | 0.9574 | 0.2171 - 0.8489i | 0.8762 |

6. All symbol error rates (SER) should be 0 because there is no noise.

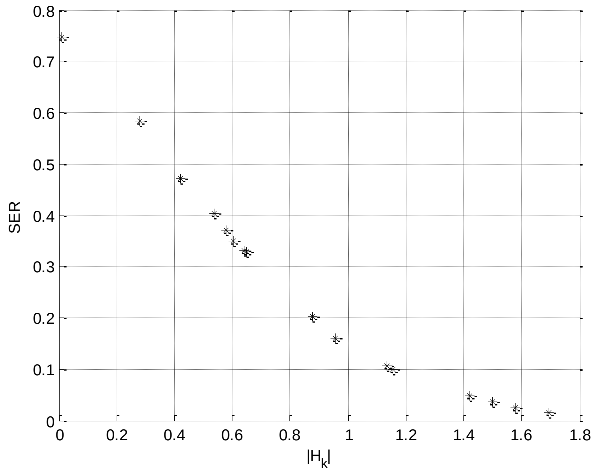
7. In this question, the channel noise is generally non-zero. $w[n]$ is now i.i.d. complex-valued Gaussian noise. Its real and imaginary parts are i.i.d. zero-mean Gaussian with variance $N_0$ /2 where $N_0$ = 1.

**Solution**

a.

| | $\rho = 0.5$ | | $\rho = 0.99$ | |
|---|---|---|---|---|
| Ch # $k$ | $\lvert H_k\rvert$ | SER | $\lvert H_k\rvert$ | SER |
| 0 | 0.5774 | 0.3631 | 0.0100 | 0.7487 |
| 1 | 0.6403 | 0.3316 | 0.5380 | 0.3919 |
| 2 | 0.6047 | 0.3601 | 0.4227 | 0.4736 |
| 3 | 0.6508 | 0.3292 | 0.2787 | 0.5750 |
| 4 | 1.1339 | 0.1032 | 1.1547 | 0.0973 |
| 5 | 1.4999 | 0.0328 | 1.6927 | 0.0145 |
| 6 | 1.4196 | 0.0423 | 1.5773 | 0.0235 |
| 7 | 0.9574 | 0.1684 | 0.8762 | 0.2028 |

b. If you try to plot $|H_k|$ vs. SER, you should get something similar to the plot below.




So, $|H_k|$ and SER go in the opposite directions. The channel that has large value of $|H_k|$ will have very good SER performance; that is it will has low SER.
Furthermore, the SER of ch#0 when $H_0$ is about 0 should be very close to 0.75. This is because the channel gain destroys almost all the information contained in the received signal. Hence, the ML detector will be correct with probability 0.5 for each dimension. The complex number (QPSK symbol) has two dimensions. Hence, the chance that it will be decoded <u>correctly</u> is $0.5 \times 0.5 = 0.25$.
If you are good at digital communications, you may check that the SER is given by

$$2p - p^2 \text{ where } p = Q\left( |H_k| \sqrt{\frac{2}{N_0}} \right).$$