Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

ECS 455: Problem Set 3

Semester/Year: 2/2012

Course Title: Mobile Communications

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Due date: Jan 14, 2013 (Monday), 10:30AM

1. Complete the following M/M/m/m description with the following terms:

(I) Bernoulli	(II) binomial	(III) exponential
(IV) Gaussian	(V) geometric	(VI) Poisson

The Erlang B formula is derived under some assumptions. Two important assumptions are (1) the call request process is modeled by a/an ______(A)________ process and (2) the call durations are assumed to be i.i.d. ______(B)________ random variables. For the call request process, the times between adjacent call requests can be shown to be i.i.d. ______(C)________ random variables. On the other hand, if we consider non-overlapping time intervals, the numbers of call requests in these intervals are ______(D)________ random variables.

In order to analyze or simulate the system described above, we consider slotted time where the duration of each time slot is small. This technique shifts our focus from continuous-time Markov chain to discrete-time Markov chain. In the limit, for the call request process, only one of the two events can happen during any particular slot: either (1) there is one new call request or (2) there is no new call request. When the slots are small and have equal length, the numbers of new call requests in the slots can be approximated by i.i.d. _____(E)_________ random variables. In which case, if we count the total number of call requests during n slots, we will get a/an

_____(F)_________ random variable because it is a sum of i.i.d. _____(E)_________ random variables.

When we consider a particular time interval *I* (not necessarily small), the number of slots in this interval will increase as the slots get smaller. In the limit, the number of call requests in the time interval *I* which we approximated by a _____(F)_________ random variable before will approach a/an ______(D)________ random variable.

Similarly, if we consider the numbers of slots between adjacent call requests, these number will be i.i.d. ______(G)________ random variables. These random variables can be thought of as discrete counterparts of the i.i.d. ______(C)________ random variables in the continuous-time model.

Some term(s) above is/are used more than once. Some term(s) is/are not used.

2. (**Markov Chain**) The Land of Oz is blessed by many things, but not by good weather. They never have two nice days in a row. If they have a nice day, they are just as likely to have snow as rain the next day. If they have snow or rain, they have an even chance of having the same the next day. If there is change from snow or rain, only half of the time is this a change to a nice day. [Grinstead and Snell, Ex 11.1][Kemeny, Snell, and Thompson, 1974]
    a. Draw the Markov chain corresponding to how the weather in the land of Oz changes from one day to the next.
       Hint: This Markov chain will have three states: nice (N), snow (S), and rain (R).
    b. Find the steady-state probabilities.
    c. Suppose it is snowing in the land of Oz today. Estimate the chance that it will be a nice day next year (365 days later)?

3. Consider a system which has 3 channels. We would like to find the blocking probability via the Markov chain method. For each of the following models, **draw the Markov chain** via discrete time approximation. Don't forget to indicate the transition probabilities on the arrows. Assume that the duration of each time slot is 1 millisecond. Then, find (1) the **steady-state probabilities** and (2) the long-term **call blocking probability**.
   a. ***Erlang B*** model: Assume that the *total* call request rate is 10 calls per hour and the average call duration is 12 mins.
   b. ***Engset*** model: Assume that there are 5 users. The call request rate for *each* user is 2 calls per hour and the average call duration is 12 mins.
   c. ***Engset*** model: Assume that there are 100 users. The call request rate for *each* user is 0.1 calls per hour and the average call duration is 12 mins.

4. (Difficult) In class we have seen that the steady-state probabilities for the Engset model are given by

$$p_i = \frac{\binom{n}{i} A_u^i}{\sum_{k=0}^{m} \binom{n}{k} A_u^k} = \frac{\binom{n}{i} A_u^i}{z(m,n)}, \quad 0 \le i \le m,$$

where $z(m,n) = \sum_{k=0}^{m} \binom{n}{k} A_u^k$.

   a. Express p_m (time congestion) in the form $p_m = 1 - \frac{z(m-c,n)}{z(m,n)}$.

      What is the value of c?
      Hint: c is an integer.

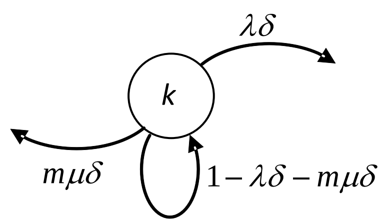
   b. The blocked call probability is given by $P_b = \frac{(n-m)\binom{n}{m} A_u^m}{\sum_{k=0}^{m} \binom{n}{k}(n-k) A_u^k}$ which can be rewritten in the form $P_b = 1 - \frac{z(m-c_1, n-c_2)}{z(m-c_3, n-c_4)}$.

      Find c_1, c_2, c_3, c_4.
      Hint: c_1, c_2, c_3, c_4 are all integrers.

   c. Suppose $m = n-1$. Simplify the expression for P_b.

      Hint: Your answer should be of the form $(g(A))^m$ for some function *g* of *A*.

5. Consider another modification of the M/M/m/m (Erlang B) system. (There are infinite users) Assume that there is a queue that can be used to hold all requested call which cannot be immediately assigned a channel. This is referred to as an M/M/m/∞ or simply M/M/m system. We will define state *k* as the state where there are k calls in the system. If $k \le m$, then all of these calls are ongoing. If $k > m$, then *m* of them are ongoing and k-m of them are waiting in the queue.
   Assume that the total call request process is Poisson with rate λ and that the call durations are i.i.d. exponential random variables with expected value $1/\mu$.
   Also assume that $\frac{\lambda}{\mu} < m$.
   a. **Draw the Markov chain** via discrete time approximation. Don't forget to indicate the transition probabilities on the arrows.
      Hint: there are infinite number of states. The transition probabilities for state *k* which is < *m* are the same as in the M/M/m/m system. For $k \ge m$, the transition probabilities are given below:



      Explain why the above transition probabilities make sense.
   b. Find the **steady-state probabilities**
   c. Find the long-term **delayed call probability** (the probability that a call request occurs when all *m* channels are busy and thus has to wait).
      Hint: This will be a summation of many steady-state probabilities. When you simplify your answer, the final answer should be

$$\frac{A^m}{A^m + m!\left(1-\frac{A}{m}\right)\sum_{k=0}^{m-1}\frac{A^k}{k!}}.$$